Multiple Objective Optimisation of a Mass Transfer in Stirred Tank Bioreactors

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Summary: In the present study we used the concept of Pareto for solving multiple objective optimisation of the mass transfer in a stirred tank bioreactor. The model of the mass transfer is formulated as a general multiple objective optimization problem. By using an assigned membership function for each of the objectives, the general multiple objective optimization problem can be converted into a maximizing decision problem. In order to obtain a global solution, a Price method is introduced to solve the maximizing decision problem. The policy consists of constructive and regime bioreactor parameters. After this multiple optimization, the performance of the mass transfer, aeration and mixing are improved.

Keywords: Multiple objective optimization problem, Mass transfer, General multiple objective optimization, Price method for optimization

1. INTRODUCTION

Multiple objective optimization is a natural extension of the traditional optimization of a single objective function. If the multiple objective functions are commensurate, minimizing single objective function, it is possible to minimize all criteria and the problem can be solved using traditional optimization techniques. On the other hand, if the objective functions are incommensurate, or competing, then the minimization of one objective function requires a compromise with another objective function. The competition between multiple objective functions is a key distinction between multiple objective optimization and traditional single-objective optimization [2, 11, 13, 14].

In previous investigations for the optimisation of the mass-transfer, aeration and mixing in the bioreactor was used single optimisation of the volumetric oxygen mass-transfer coefficients and gas-liquid transition [4, 5]. For solving the single objective optimization

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problem the theory of fuzzy sets and combined algorithm combines the method of random search and method of the theory of fuzzy sets have been used [7, 8].

In this study, a fuzzy procedure has been used for finding the optimal values of the design and regime parameters of a bioreactor to improve the performance of the mass-transfer, aeration and agitation.

2. MATERIALS AND METHODS

2.1. Model of mass-transfer in the stirred tank bioreactor

The model is based on the following assumptions [3]: the change of the gas phase (GP) concentration C_G is described by a diffusion model at a steady-state condition; the change the liquid phase (LP) concentration C_L is described by a perfectly mixed model; the balance of the oxygen concentration C^* is determined as an average value in liquid height into the bioreactor.

The decision model in dimensionless variables has the look [7, 8]:

$$X(\eta,t) = A_0 \exp(r_1\,\eta) + B_0 \exp(r_2\,\eta) + C_0 \qquad (1)$$

$$X^*(t) = A_0 \frac{\exp(r_1)-1}{r_1} + B_0 \frac{\exp(r_2)-1}{r_2} + C_0 \qquad (2)$$

$$Y(t) = X^*(t)\left[1-\exp(-a_2\,t)\right] \qquad (3)$$

where: $X(\eta,\ t)$ – dimensionless oxygen concentration in GP, $X(\eta,t) = C_G(z,t)/C_G^0$; $X^*(t)$ – dimensionless balance of the oxygen concentration, $X^*(t) = C^*(t)/C_G^0$; $Y(t)$ – dimensionless oxygen concentration in LP, $Y(t) = C_L(t)\,m_L / C_G^0$; m_L – Henry law constants; η – dimensionless coordinate, $\eta = z/u_8$; u_8 – level of liquid in the bioreactor, m; z – stirrer distance from bottom of bioreactor; C_G^0 – oxygen concentration in the air, $C_G^0 \cong 0.24$ kg·m^{-3}.

The initial and boundary conditions for the dimensionless variables are given as follows:

$$t = 0 : Y(0) = 0; \quad X(\eta,0) = 1; \quad 0 \le \eta \le 1$$

$$\eta = 0: 1 = X - \frac{1}{Pe}\frac{dX}{d\eta}; \quad \eta = 1: \ \frac{dX}{d\eta} = 0$$

The constants in the model are determined by the following relations: $A_0 = a_3 B_0$, $B_0 = \exp(-a_2 t)/a_4$, $C_0 = 1 - a_4 B_0$, where [5, 7, 8]:

$$r_{1,2} = 0.5Pe \pm \sqrt{(0.5Pe)^2 + a_1}\ , \ Pe = W_G u_8 / D_L$$

$$a_1 = \frac{k_l a\, u_8}{\varepsilon_G m_L W_G}\ , \ a_2 = \frac{k_l a}{\varepsilon_L}\ , \ \varepsilon_L = 100 - \varepsilon_G\ , \ \varepsilon_G = 0.2\left(\frac{P_G}{V}\right)^{0.7} W_G^{0.2}$$

$$a_3 = -(r_2 / r_1) \exp\left(-2\sqrt{(0.5\,Pe)^2 + a_1}\right)$$

$$a_4 = 1 + a_3 - (a_3 r_1 + r_2)/Pe\ , \ W_G = \frac{4\,u_7}{\pi D^2}\ , \ \mathrm{Re} = \rho u_5^2 u_6 / \nu$$

$$\varepsilon_G = 0.2\,(P_G / V)^{0.7}\, W_G^{0.2}\ , \ P_G = 0.21\,(u_7 / (u_6 u_5^3))^{-0,1} P_L^{0.8}$$

$$V = 0.25\pi D^2 u_8\ , \ P_L = \begin{cases} for\ u_2 = 0 : 179.5\,\rho\, u_6^3 u_5^5\ \mathrm{Re}^{-0.40} \\ for\ u_2 > 0 :\ 60.9\,\rho u_6^3 u_5^5\, \mathrm{Re}^{-0.18} \left(u_2 / u_5\right)^{-0.23} \end{cases}$$

where Pe – Peclet number, [–]; ε_G and ε_L – volume fraction of gas and liquid in the bioreactor, [vol. %]; $k_l a$ – volumetric mass transfer coefficient, [s^{-1}]; D_L – dispersion coefficient, [$m^2 \cdot s^{-1}$]; W_G – reduce gas flow rate, [$m \cdot s^{-1}$]; P_G – power input with aeration, [W]; P_L – power input without aeration, [W]; Re – Reynolds number, [–]; ν – liquid dynamic viscosity, [Pa.s]; ρ – liquid density, [$kg \cdot m^{-3}$]; V – volume, [m^3]; D – bioreactor diameter, [m]; ρ_G – gas density, ρ_G = 1.141 [$kg \cdot m^{-3}$]; u_2 – eccentricity of impeller, [m]; u_5 – impeller diameter, [m]; u_6 – rotation speed, [s^{-1}]; u_7 – gas flow rate, [$m^3 \cdot s^{-1}$]; u_8 – level of liquid in the bioreactor, [m].

The volumetric coefficient $k_l a$ is determined by the following regression model [4-7]:

$$\begin{aligned} k_l a = 136.5 + 8.9u_1 + 13.1u_3 + 7.3u_2u_4 + 6.1u_3u_4 - \\ - 7.9u_1^2 - 34.4u_2^2 - 9.3u_3^2 + 23.5u_4^2 \end{aligned} \quad (4)$$

2.2. System Constraints

Nearly all engineering processes will have physical constraints. The dimensionless oxygen concentration in GP, the dimensionless mean concentration, and the dimensionless oxygen concentration in LP must be positive for all time; otherwise, an unrealistic solution in the optimization problem would be obtained, that can described with:

$$g_1 = -X(\eta, t) \le 0 \tag{5}$$

$$g_2 = -X^*(t) \le 0 \tag{6}$$

$$g_3 = -Y(t) \le 0 \tag{7}$$

If the constraints in (5)–(7) are not included in the optimization problem, unrealistic predicted values may be found.

3. MULTIPLE OBJECTIVE OPTIMISATION PROBLEM

3.1. Formulation of the multiple objective optimisation problem

The multiple objective optimisation problem (MOOP) is to find optimal angle of the blades of the impeller – u_1, eccentricity of impeller – u_2, number of impeller – u_3, width of the baffle assembly – u_4, impeller diameter – u_5, rotation speed – u_6, gas flow rate – u_7, level of liquid in the bioreactor – u_8, final time – u_9, and u_{10} – stirrer distance from bottom of bioreactor such that the mass transfer, aeration and mixing processes are greater than or equal to a threshold value.

The control variables $u_1 \div u_4$ are coded in the intervals from –1 to +1. The rest variables are satisfied in the following intervals:

$(40 \le u_5 \le 60)$ [mm], $(200 \le u_6 \le 1200)$ [min^{-1}],
$(50 \le u_7 \le 300)$ [$\text{l}\cdot\text{h}^{-1}$], $(100 \le u_8 \le 175)$ [mm].

The mass transfer, aeration and mixing processes have been characterised by the following based indexes [3]:

$$\max_{\mathbf{u}} J_1 = C_G^0 \frac{k_l a\, V}{\rho_G\, u_7} \frac{[X^*(t_f) - Y(t_f)]}{X(\eta, t_f)} \tag{8}$$

$$\max_{\mathbf{u}} J_2 = \varepsilon_L \tag{9}$$

$$\min_{\mathbf{u}} J_3 = \left(P_L + P_G\right)/V \tag{10}$$

where: t_f – final time, [s].

The first objective function corresponds to the index characterized mass transfer gas-liquid. The second objective function corresponds to the volume fraction of the liquid in the bioreactor. The last objective function corresponds to the summary relative power inputs.

3.2. Solution of the MOOP

Assume that the decision making (DM) has fuzzy goals for each of the objective functions in (8)-(10). The MOOP (8)-(10) is now extended to the general multiple objective optimization problem (GMOOP) given as:

$$\mathit{fuzzy} \max_{\mathbf{u}} J_1 = C_G^0 \frac{k_l a\ V}{\rho_G\, u_7} \frac{[X^*(t_f) - Y(t_f)]}{X(\eta, t_f)} \tag{11}$$

$$\mathit{fuzzy} \max_{\mathbf{u}} J_2 = \varepsilon_L \tag{12}$$

$$\mathit{fuzzy} \min_{\mathbf{u}} J_3 = \left(P_L + P_G\right)/V \tag{13}$$

The membership function of (11) and (12) has the type

$$\mu_k(J_k) = \begin{cases} 0; & J_k < J_k^L, \quad k = 1,2 \\ \dfrac{J_k - J_k^L}{J_k^U - J_k^L}; & J_k^L \le J_k \le J_k^U \\ 1; & J_k > J_k^U \end{cases} \tag{14}$$

where J_k^{L} or J_k^{U} represents the value of J_k, such that the grade of the membership function $\mu(J_k)$ varies from 0 to 1.

The membership function for minimizing goals of (13) is expressed as:

$$\mu_3(J_3) = \begin{cases} 1; & J_3 \le J_3^L \\ \dfrac{J_3 - J_3^L}{J_3^U - J_3^L}; & J_3^L < J_3 < J_3^U \\ 0; & J_3 \ge J_3^U \end{cases} \tag{15}$$

where J_3^L or J_3^U represents the value of J_3 such that the grade of the membership function $\mu(J_3)$ is from 1 to 0.

As a result, the DM considers the fuzzy objective function such as J_1 and J_2 should be substantially greater than or equal to a threshold interval $[J_k^L \ J_k^U]$, $k = 1, 2$. The third and four, goals should be substantially less than or equal to the assigned threshold interval $[J_3^L \ J_3^U]$. The membership function for each of the objective functions is described in Fig. 1.

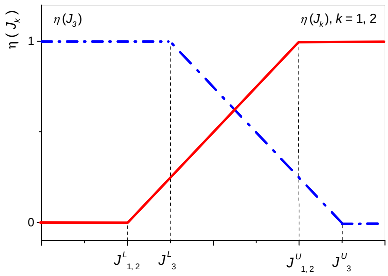

Fig. 1 Assigned membership function for each of the objective functions

Having elicited the membership functions for each of the objective functions, the GMOOP (11)-(13) can be converted into the fuzzy multiple objective optimization problem (FMOOP) by [13]:

$$\min_{\mathbf{u}\in\Omega} \left[\mu_1(J_1) \ \ \mu_2(J_2) \ \ \mu_3(J_3)\right]^T \tag{16}$$

By introducing a general aggregation function $\mu_D(J_k)$, a fuzzy multiple objective decision making problem (FMODMP) or maximizing decision problem can be defined by

$$\max_{\mathbf{u}\in\Omega} \mu_D \tag{17}$$

Several aggregation functions have been used in the standard fuzzy nonlinear programming [10]. In this study, the fuzzy decision or minimum operator of Bellman and Zadeh [1] is selected as the aggregation function by:

$$\mu_D = \min_k \{ \mu_k(J_k), k = 1, \ldots, 4 \}$$

Fundamental to the MOOP in (8)-(10) is the Pareto optimal concept, and thus the DM must select a compromise solution from many Pareto optimal solutions. The relationships between the optimal solutions of the FMODMP and the Pareto optimal concept of the MOOP can be characterized by the following theorem [13]:

Theorem 1. If $\mathbf{u}^*$ is a unique optimal solution to the FMODMP in (17), then $\mathbf{u}^*$ is a Pareto optimal solution to the MOOP in (8)-(10).

This theorem serves to guarantee that the unique optimal solution of the FMODMP is a Pareto solution to the crisp MOOP (8)-(10). The statement of this theorem does not guarantee that the unique optimal solution to (17) is a Pareto solution to the GMOOP (11)-(13).

Definition 1. If $\mathbf{u}^* \in \Omega$ is said to be an M-Pareto optimal solution to GMOOP if and only if no other $\mathbf{u} \in \Omega$ exists there, such that $\mu_k(J_k(\mathbf{u})) \geq \mu_k(J_k(\mathbf{u}^*))$ for all k and $\mu_j(J_j(\mathbf{u})) \neq \mu_j(J_k(\mathbf{u}^*))$ for at least one j.

Note that the set of Pareto optimal solutions is a subset of the set of M-Pareto optimal solutions, as observed from Definitions 1 and (14)-(15). Here M refers to membership. Using the concept of M-Pareto optimality, the fuzzy version of Theorem 1 can be obtained under slightly different conditions.

Theorem 2. If $\mathbf{u}^*$ is a unique optimal solution to the FMODMP (17), then $\mathbf{u}^*$ is an M-Pareto optimal solution to the GMOOP (11)-(13).

Theorem 2 is used to guarantee that the unique optimal solution of the maximizing decision problem (17) is an M-Pareto optimal solution of the fuzzy problems (11)-(13). The key point for using this theorem is to find a unique optimal solution of the problem (17). A global optimization method must be employed to determine such a unique solution.

An interactive programming algorithm is introduced in this study and it is listed below to find a satisfactory solution to the GMOOP:

1. Assign the threshold intervals $\left[J_k^L\ J_k^U\right]^r$.
2. Elicit a membership function $\mu(J_k)$ from the DM for each of the objective functions.
3. Solve the maximizing decision problem (17). Let $\left(\mathbf{u}^r, \mu_k^r(J_k)\right)$ be the M-Pareto optimal solution to the GMOOP.
4. If the DM is satisfied with the current levels of $\mu_k^r(J_k)$, the current M-Pareto optimal solution $\left(\mathbf{u}^r, \mu_k^r(J_k)\right)$ is the satisfactory solution for the DM. Otherwise, classify the objectives into three groups based on the DM's preference, including:

   (a) class of the objectives that the DM wants to *improve*,
   (b) a class of the objectives that the DM may possibly agree to *relax*, and
   (c) a class of the objectives that the DM *accepts*.

   The index set of each class [13] is represented by $\mathbf{I}^r$, $\mathbf{R}^r$, and $\mathbf{A}^r$, respectively. The new threshold intervals $\left[J_k^L\ J_k^U\right]^{r+1}$ are reassigned in such a way that $\left[J_k^L\ J_k^U\right]^r \subset \left[J_k^L\ J_k^U\right]^{r+1}$ for any $k \in \mathbf{I}^r$, $\left[J_k^L\ J_k^U\right]^{r+1} \subset \left[J_k^L\ J_k^U\right]^r$ for any $k \in \mathbf{R}^r$, and $\left[J_k^L\ J_k^U\right]^{r+1} = \left[J_k^L\ J_k^U\right]^r$ for any $k \in \mathbf{A}^r$. Then repeat Step 2.

Here, it should be emphasized that any improvement for one of the objective functions can be achieved only at the expense of at least one of the other objective functions.

4. RESULTS AND DISCUSSION

Now, the maximizing decision problem (17) can be solved by Price method for searching of global extremum [9].

Since the physical constraints in (5)-(7) are included in the optimization problem, the penalty function method is used to handle the system constraints in fuzzy optimizations. The general function used in fuzzy optimizations is, therefore, defined as

$$\max_{\mathbf{u}} J = \mu_D - \sum_{i=1}^{3} \lambda_i \int_0^{t_f} \langle g_i \rangle_+^2 \, dt \qquad (18)$$

The integration of the square penalty functions in (18) is used to cover the state variables on the whole time domain.

The Price method is well known from the literature [12]. The algorithm of Price method was written in FORTRAN 77. All computations were performed on an Intel 1.8 GHz computer using Microsoft Windows XP Pro Edition operating system.

The obtained initial and optimal values of the control variables and criteria are shown in Table 1, for m_L = 33.0, D_L = 52.08×10^{-6} [$m^2 \cdot s^{-1}$], $\eta = 0.5$, and $t_f = 30$ [s].

Table 1. Optimisation results

Variables	u_1 0	u_2 mm	u_3 num.	u_4 mm	u_5 mm	u_6 min^{-1}	u_7 $l \cdot h^{-1}$	u_8 mm	D mm
Initial values	90	0.0	1	12	57	600	60	114	114
Optimized values	86	1.0	2	13	43	276	53	162	
Variables	W_G $mm \cdot s^{-1}$	P_L W	P_G W	k_la h^{-1}	Pe -	V l	J_1	J_2 Vol. %	J_3 $W \cdot l^{-1}$
Initial values	1.63	1.8	1.50	80	3.6	1.2	4.9	93.4	2.83
Optimized values	1.44	1.2	0.90	130	4.5	1.7	8.4	96.2	1.30

The obtained results (Table 1) show a significant increase of the values of J_1 and J_2. The index characterized mass transfer gas-liquid has increased with more than 70% (J_1 = 4.9 – before and J_1 = 8.4 after optimization). The index characterized volume fraction of the liquid in the bioreactor has increased with more than 2.9% (J_2 = 93.4% – before and J_2 = 96.2 after optimization).

The summary relative power (Table 1) has decreased with more than 110% (J_3 = 2.83 – before and J_3 = 1.3 – after optimisation).

Basic mass transfer index k_la has increased its value (Table 1) with more than 60% (k_la = 80 h^{-1} – before and k_la = 130 h^{-1} – after optimisation).

The developed optimisation of the gas-liquid transition (criteria J_1) in dependence on the considered constructive and regime bioreactor parameters, maximises the efficiency of the apparatus, applied to aerobic bioprecesses.

5. CONCLUSIONS

1. This study discusses a multiple objective optimization of the mass transfer in the stirred tank bioreactor planning problem. Many of the multiple objective optimization problems in the real world take place in an environment in which the goals, the constraints, and the consequences of possible actions are not precisely known. To quantitatively deal with imprecision, the problem in a fuzzy environment is introduced in this study to handle these imprecise goals and constraints. Such fuzzy multiple objective optimal control problems are converted into a maximizing decision problem through the subjective membership functions for each of the objective functions. The optimal solution for each of the membership functions is denoted as the degree of satisfaction with the assigned threshold requirements.
2. A simple guideline is presented in the interactive programming procedures in order to find a satisfactory solution to the general multiple objective optimization problem. In order to obtain a global optimal solution, a Price method is introduced to solve the maximizing decision problem.
3. The general conclusion after optimisation of the constructive bioreactor parameters is that the impeller with fewer diameters should be used (Table 1, parameter u_5). It will reduce the diameter of the impeller shaft. The rest parameters do not cause any appreciable change in the equipment.
4. The determined optimal values of the constructive parameters show that the offered laboratory bioreactor with sliding bearings and disc magnetic coupling demonstrates a good mass-transfer, aeration and agitation indexes.

REFERENCES

1. Bellman R., L. Zadeh, Decision making in a fuzzy environment, *Manag. Sci.*, 1970, 17(4), B141–B164.
2. Chen Y., F.-S. Wang, Crisp and fuzzy optimization of a fed-batch fermentation for ethanol production, *Industrial & Eng. Chem. Research*, 2003, 42, 6843–6850.
3. Jenne M., M. Reuss, Fluid Dynamic Modelling and Simulation of Gas-Liquid Flow in Baffled Stirred Tank Reactors, Récents Progrès en Génie des Procédés (Mixing IX: Two phase Systems), 1997, 11(52), 201–208.
4. Petrov M., A Combined algorithm for optimization: An application for optimization of gas-liquid transition in stirred tank bioreactors, *Bioautomation*, 2005, 3, 10–18.
5. Petrov M., Fuzzy optimization of mass transfer in stirred tank bioreactors, *Biotechnology & Biotechnological Equipment*, 1998, 12(1), 120–124.
6. Petrov M., Investigation of power characteristics of small scale stirred tank bioreactor, *Biotechnol. & Biotechnol. EQ.*, 2005, 19(1), 205–209.
7. Petrov M., St. Tzonkov, Modelling of mass transfer and optimization of stirred bioreactors, *Bioprocess Engineering*, 1999, 21(1), 61–63.
8. Petrov M., T. Ilkova, St. Tzonkov, U. Viesturs, Modeling, optimization and optimal control of small scale stirred tank bioreactors, *Bioautomation*, 2004, 1, 67–82.
9. Price W., Controlled Random Search Procedure for Global Optimization, Report 75–23, University of Leicester, Eng. Dept., 1975.
10. Sakawa M., Fuzzy Sets and Interactive Multiobjective Optimization, Plenum Press: New York, 1993.
11. Sendín O., J. Vera, T. Nestor, Model based optimization of biochemical systems using multiple objectives: A comparison of several solution strategies, *Mathematical and Computer Modelling of Dynamical Systems*, 2006, 12(5), 469–487.
12. Stoyanov St., Optimization Methods and Algorithms, Technique, Sofia, 1990 (in Bulgarian)

13. Wang F.-S., C.-H. Jing, G. Tsao, Fuzzy-Decision-Making Problems of Fuel Ethanol Production Using a Genetically Engineered Yeast, *Industrial and Engineering Chemistry Research*, 1998, 37, 3434–3443.
14. Zhou Y. H., N. J. Titchener-Hooker, The Application of a Pareto Optimisation Method in the Design of an Integrated Bioprocess, *Bioprocess and Biosystems Engineering*, 2003, 25, 349–355.