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Application of K-Nearest Neighbor Rule in the Case of Intuitionistic Fuzzy Sets for Pattern Recognition

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Summary: In the present paper an algorithm based on the k-nearest neighbors (*KNN*) rule modified for the case of intuitionistic fuzziness is proposed. The algorithm calculates the degrees of membership, non-membership and indeterminacy for each new element that needs to be classified. The choice of the *KNN* rule is due to the high precision of the method in decision making for pattern recognition problems, while the apparatus of the intuitionistic fuzzy sets is used to describe more adequately the considered objects and allows for pattern recognition with non-strict membership of the patterns.

Keywords: Intuitionistic fuzzy sets, k-nearest neighbors rule, Pattern recognition

1. INTRODUCTION

Intuitionistic fuzzy sets (*IFS*) were introduced by Atanassov in 1983 [1]. They are an extension and generalization of the fuzzy sets (*FS*) which Zadeh introduced in [2]. *FS* and subsequently *IFS* are mainly used to address problems with imprecise or incomplete data. An *IFS* is usually denoted as an ordered triple, e.g. $\langle x, \mu_A(x), \nu_A(x) \rangle$, where x is an element of some universal set E, A is a subset of E and $\mu_A(x)$, $\nu_A(x)$ denote the degree of membership and non-membership of the element x to the set A. The mappings

$$\mu : E \to [0,1], \ \ \nu : E \to [0,1]$$

are chosen so that for all it is fulfilled that:

$$\mu_A(x) + \nu_A(x) \leq 1$$

By choosing $\nu_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x)$, we obtain the *FS*.

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In the case of *IFS* the difference $\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x)$ is called degree of indeterminacy and represents the level of uncertainty associated with the information. One of the most used geometric interpretations is the interpretation triangle introduced in [3]. It is shown on Fig. 1.

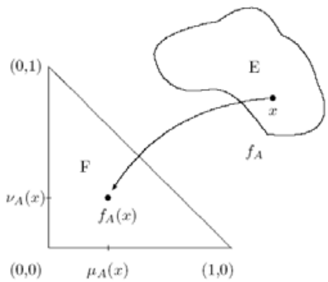


Fig. 1 The interpretation triangle

IFS have been successfully used to improve pattern recognition (see e.g. [4, 5, 6, 7]).

The *KNN* classifier has been used in different areas with great success [8, 9, 10]. *KNN* classification algorithm tries to find the K nearest neighbors of a given unclassified pattern x_0 and uses a majority vote to determine the class label of x_0. An example is shown on Fig.2.

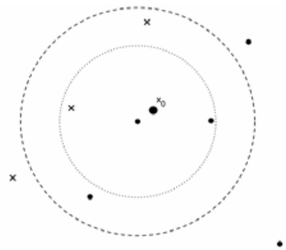


Fig. 2 In the inner circle the case *k*=3 has been shown, the bigger circle corresponds to *k*=5

This simple and easy-to-implement method can still yield competitive results even compared to the most sophisticated machine learning methods. Because of this the *KNN* rule is one of the most used and precise classification method based on distance functions.

The high performance of the two approaches led to the idea of their combination to develop an improved recognition rule.

2. ALGORITHM

In the present work we propose an algorithm which determines the degree of membership, degree of non-membership, degree of indeterminacy in the sense of *IFS* using the *KNN* rule. For clarity we will consider the case of two classes – class ω_1 and class ω_2. The patterns belonging to class ω_1 we will denote by x_i^1, the patterns belonging to class ω_2 we will denote by x_i^2.

Description of the algorithm:

Step 1: We identify etalons (*by etalon here and further we denote a typical or representative pattern for the given class*) for each of the considered classes. Let them be ε_1 and ε_2 for class ω_1 and class ω_2, respectively.

Step 2: We find the closest to ε_1 pattern x_i^2 from class ω_2. We will denote the distance between ε_1 and x_i^2 by r_1. We construct a circle $C_{\min}^1$ with center ε_1 and radius r_1. We make an analogous construction for the class ω_2. Obviously, the disks bounded by the so-constructed circles contain only images from the respective class (e.g. – in the disk bounded by $C_{\min}^1$ there are only images from class ω_1). The degrees of membership for the patterns in these areas are $\mu_i^1 \stackrel{\text{def}}{=} 1$ and $\nu_i^2 \stackrel{\text{def}}{=} 0$ (respectively $\mu_i^2 \stackrel{\text{def}}{=} 1$ and $\nu_i^2 \stackrel{\text{def}}{=} 0$ for class ω_2).

Step 3: We find the furthest from ε_1 pattern x_i^1 from class ω_1. We will denote the distance between ε_1 and x_i^1 by R_1. We construct a circle $C_{\max}^1$ with center ε_1 and radius R_1. We make an analogous construction for class ω_2. Obviously, each of the disks bounded by the so-constructed circles contains all patterns (and not only them) of the respective class (e.g. in the disk bounded by $C_{\max}^1$, there are all images from class ω_1, as well as images from ω_2).

Step 4: Let $\varepsilon_1\varepsilon_2 \cap C_{\min}^1 = p^1$ and $\varepsilon_1\varepsilon_2 \cap C_{\min}^2 = p^2$. Let m be (preliminary selected) natural number $s = \frac{\text{dist}(p^1, p^2)}{m}$. We construct the concentric circles $C_i^1(\varepsilon_1, r^1 + is)$ and $C_i^2(\varepsilon_2, r^2 + is)$ (see Fig. 3.)

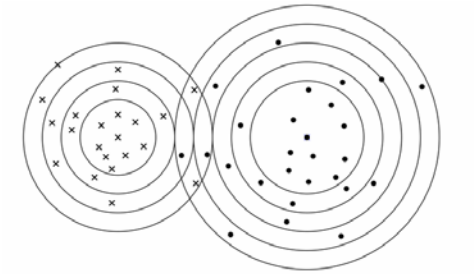


Fig. 3 The result of **Step 4**

Step 5: In all so-generated annuli V from both classes we find the number of contained patterns and let

$$n_{\min} = \begin{cases} 2 \text{ if } \min n_i < 2; \\ \min n_i \text{ if } \min n_i \geq 2 \geq 2. \end{cases}$$

We choose an odd $k < n_{min}$. For every pattern x_i^δ (where δ denotes the class) we determine the degree of membership by the formula:

$$\mu(x_i^{\delta_j}) = \frac{n_{\delta_j}}{n_{\delta_j} + n_{\overline{\delta_j}} + 1}$$

where $\overline{\delta}$ denotes the other class and δ_j, $\overline{\delta_j}$ refer to the j-th annuli for the class δ.

Step 6: For every x_i^δ we determine the degree of non-membership by the formula:

$$\nu(x_i^\delta) = \frac{n_{\overline{\delta}}}{n_\delta + n_{\overline{\delta}} + 1}$$

Step 7: For every x_i^δ we determine the degree of indeterminacy by the formula:

$$\pi(x_i^\delta) = 1 - \mu(x_i^\delta) - \nu(x_i^\delta)$$

Step 8: For every annulus we determine the aggregated values for the degrees following the formulas:

$$\mu(V_i) = \left(1 + \frac{is}{R_\delta}\right)^{-1} \frac{1}{t} \sum_{i=1}^{t} \mu\left(x_i^{\delta_j}\right)$$

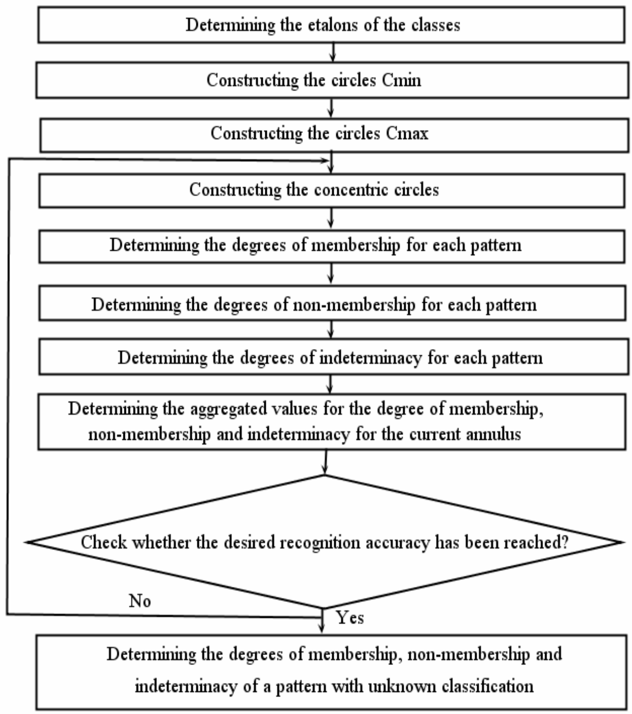
$$\pi(V_i) = \frac{1}{t} \sum_{i=1}^{t} \pi\left(x_i^{\delta_j}\right)$$

$$\nu(V_i) = 1 - \mu(V_i) - \pi(V_i)$$

Step 9: We check whether the desired recognition accuracy has been reached. If "yes" – we go to Step 10, if "no" – return to Step 4 and select another value for *m*.

Step 10: We determine the degrees of membership, non-membership and indeterminacy of a pattern with unknown classification. To each pattern with unknown classification we assign the degrees of the annulus in which it is contained.

Flowchart of the algorithm



3. CONCLUSION

The proposed algorithm is appropriate for solving pattern recognition problems in medicine, since it combines the precision of the *KNN* rule with the apparatus of the *IFS*, which reflects the fact that one and the same illness develops differently in different patients. This particularity of the data obtained in medical research is accounted for by the different degrees of membership of the patient to a given class corresponding to the specific disease.

The accuracy of the algorithm depends on two parameters – *k* (the number of nearest neighbors in the *KNN* rule) and *s* (the width of the annuli). For every particular problem these parameters are adjusted until a maximum precision is achieved, which makes the algorithm flexible and suitable for a wide range of applications.

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