Uncertainty Estimator based Nonlinear Feedback Control for Tracking Trajectories in a Class of Continuous Bioreactor

Maria Isabel Neria-González[1], Ricardo Aguilar-López[2*]

[1]*División de Ingeniería Química y Bioquímica*
Tecnológico de Estudios Superiores de Ecatepec
Av. Tecnológico S/N, Ecatepec
CP 55120 Edo de México, México
E-mail: ibineria@hotmail.com

[2]*Departamento de Biotecnología y Bioingeniería*
CINVESTAV-IPN
Av. Instituto Politécnico Nacional
2508, San Pedro Zacatenco, 07360 Mexico City, Mexico
E-mail: raguilar@cinvestav.mx

[*]*Corresponding author*

Received: May 28, 2014 **Accepted: December 12, 2014**

Published: April 01, 2015

Abstract: *The main goal of this work is presents an alternative design of a class of nonlinear controller for tracking trajectories in a class of continuous bioreactor. It is assumed that the reaction rate of the controlled variable is unknown, therefore an uncertainty estimator is proposed to infer this important term, and the observer is coupled with a class of nonlinear feedback. The considered controller contains a class of continuous sigmoid feedback in order to provide smooth closed-loop response of the considered bioreactor. A kinetic model of a sulfate-reducing system is experimentally corroborated and is employed as a benchmark for further modeling and simulation of the continuous operation. A linear PI controller, a class of sliding-mode controller and the proposed one are compared and it is show that the proposed controller yields the best performance. The closed-loop behavior of the process is analyzed via numerical experiments.*

Keywords: *Nonlinear feedback, Tracking trajectories, Continuous bioreactor, Sulfate-reducing system.*

Introduction

The processing of biological materials and employing biological agents such as cells, enzymes, or antibodies have been recognized since thousands of years. Bioprocess is currently involved in producing some chemical compound synthesized by a microorganism; cultivate a biomass for its utilization, extraction of its metabolites, and to degrade a pollutant. As it is known, over the past several decades, biotechnological processes have been increasingly used industrially, which is attributed to several reasons as improvement of profitability and quality in production industries, new legislative standards in processing industries, etc. Several problems arising from this industrialization are generally the same as those encountered in any processing industry, in the field of bioprocessing, almost all of the problems that are being tackled in automatic control. Thus, system requirements for supervision, control and monitoring of the processes in order to optimize operation or detect malfunctions are on the increase [1, 6]. However, in reality, few installations are provided with such systems. The bioprocess advancement is determined by the living cells capabilities and characteristics, the bioreactor performance as well as by the cultivation media composition and the main parameters evolution. The high metabolic network complexity inside the cells often determine very sophisticated, non-linear growth and product formation kinetics, with further consequences on the bioprocess behavior, but at the same time on the product quality and yield; on the one hand, lack of reproducibility of experiments and inaccuracy of measurements result not only in one or several difficulties related to selection of model structure but also in difficulties related to the concepts of structural and practical identifiability at the time of identification of a set of given parameters for estimation and control purposes.

Other difficulties also occur at the time of the validation phase of these models whose sets of parameters could have precisely evolved over course of time. These variations can be the consequence of metabolic changes of biomass or even genetic modifications that could not be foreseen and observed from a macroscopic point of view. Other important issue is the almost systematic absence of sensors providing access to measurements necessary to know the internal functioning of biological processes [8].

The majority of the key variables associated with these systems (concentration of biomass, substrates and products) can be measured only using off-line analyzers on a laboratory scale, where they exist, which are generally very expensive and often require heavy and expensive maintenance. Thus, the majority of the control strategies used in industries is very often limited to indirect control of fermentation processes by control loops of the environmental variables such as dissolved oxygen concentration, temperature, pH, etc.

The early successful application control strategy in process control is in evolution of the Proportional Integral Derivative (PID) controller and Ziegler-Nichols tuning method [21]. Till nowadays, a high percent of the controllers implemented in the process industries are PID-type [4]. However, as (i) the industrial demands, (ii) the computational capabilities of controllers and (iii) complexity of systems under control increase, so the challenge is to implement advanced control algorithms [18].

Since achievable controller performance in a model-based control scheme is dependent on the quality of the process model [11], a controller based on a model that captures events occurring at both the general considerations for control of bioreactors.

On other hand, the difficulty of implementing a feedback control is twofold. First, response of sensors tends to be slower than many of the processes they monitor. Second, the sensors are generally not available for measurement of substrate with rapid dynamics for feedback application [19]. Given the above objectives there are, broadly speaking, and two ways to design an appropriate control system. The most frequently used method is to pre-select a controller structure and then to tune the parameters of this controller so that the desired closed-loop response is obtained. This is referred to as a parameter optimized control system, the most well known example of which is probably the PID controller [2]. The other approach is the use of structure optimal control systems, where both the structure and parameters of the controller are adapted to those of the process model [3]. In practice, however, the use of the latter method is severely restricted because exact dynamic term cancellation is required in order to produce the optimal controller structure. This is usually not possible for various reasons, e.g. of the lack of an appropriate process model, non-linearities and physical constraints on the process variables. From the above, in this work is considered a class of uncertainty estimation in order to infer the unknown reaction rate of the controlled variable, this estimator is coupled with a smooth controller which is close to sliding-mode frame, where a smooth reaching law is proposed to lead to the bioreactor to stable surface, where the process is robust against some disturbances and model uncertainties (as classical sliding-mode controllers) avoiding the named chattering problem improving the closed-loop performance of the system.

Materials and methods

Conditions of culture

The bacterium Desulfovibrio alaskensis 6SR as model spices of sulfate-reducing process was described firstly by Feio [7]. The strain 6SR was isolated from a developed biofilm inside face of oil pipeline, *Desulfovibrio alaskensis* 6SR is maintained routinely in Hungate tubes with 5 mL of Postgate's medium B [9, 16]. The strain was transferred a serum bottle with 45 mL sterile Postgate's C medium in anaerobic conditions [9], and subsequently a subcultures were made. The medium of culture was prepared under anaerobic conditions and formulation previously reported [5]. The inoculum for kinetic study was cultured in 45 mL of Postgate's C medium at 37°C for 30 h until culture reached at the beginning of stationary phase. A 5 mL aliquot was taken from Postgate's C medium to inoculate 95 mL of fresh medium at 37°C. The experiment was done using two series of triplicate independent cultures; each set of triplicate cultures were inoculated with 12 hours separated each other, the experimental run time was 72 hours. One set of independent cultures were used to measure Extracellular Polymeric Substances (*EPS*) production. A culture was taken for day and the *EPS* was extracted.

Analytic methods

The bacterial growing, consuming of sulfate, and sulfide production were monitored 3 or 4 hours each, the samples were taken carefully, avoiding contact with oxygen. The bacterial growing was followed through Optical Density (OD) methodology, the OD data were transformed into dry mass (mg/mL) through a dry mass versus OD standard curve. The consuming of sulfate in the medium was measured by the turbid metric method based on barium precipitation [13]. Also the production of sulfide was measured by a colorimetric method [20]. Each measuring was done using a Thermo SCIENTIFIC GENESYS 10 uv Scaning Spectrophotometer.

The *EPS* was extracted by heat treatment and filtration. The bacterial culture bottles were opened and placed in water bath at 50°C for 15 minutes, each sample was vortexed once or twice, then the cellar suspension was passed through of a nylon membrane 0.45 μm, the filtrate was collected in 250 mL centrifuge bottle and *EPS* was then precipitated from it, adding an equal volume of cold ethanol overnight at -20°C, followed by centrifugation at 2500 × g for 10 min at 4°C (Hettich Zentrifugen UNIVERSAL 320R). The pelleted *EPS* was transferred at micro-centrifuge tube and washed in 70% (v/v) ice-cold ethanol. *EPS* was dried in oven (ECOSHEL DOV23A) at 70°C for 24 h and before dry weight was recorded.

Data analysis and mathematical kinetic model

The response variables data (biomass, sulfate, *EPS* and sulfide) of the two series of sulfate-reducing culture were analyzed and average value of each measurement point was calculated. In graphic of the average experimental data for each response variable, the hydrogen sulfide produced by bacteria showed a negative effect on growth of strain. Thereby, the specific growth rate follows an inhibition product model.

In biological system, the unstructured models are a tool to simplify the representation of bioprocess, which are essentially kinetic equation that describes the variation of substrate or product concentrations and the cell concentration as the unique biological state variable. In this work, the growing kinetic was described with a product inhibition model as following:

$$\mu = \left[\frac{\mu_{\max} S}{K_S + S + (S^2 / K_i)} \right] \left[\frac{K_P}{K_P + P} \right]. \tag{1}$$

This equation represents an unstructured kinetic model as Haldane-Boulton, where: $\mu_{\max}$ represents the maximum growing rate; K_S represents the affinity substrate; K_P corresponds to inhibition concentration product, and K_i is a kinetic constant; while S and P are substrate and product concentration, respectively. Fig. 1 is related with the model validation with experimental data, where is observed a satisfactory agreement between the predicted and experimental data.

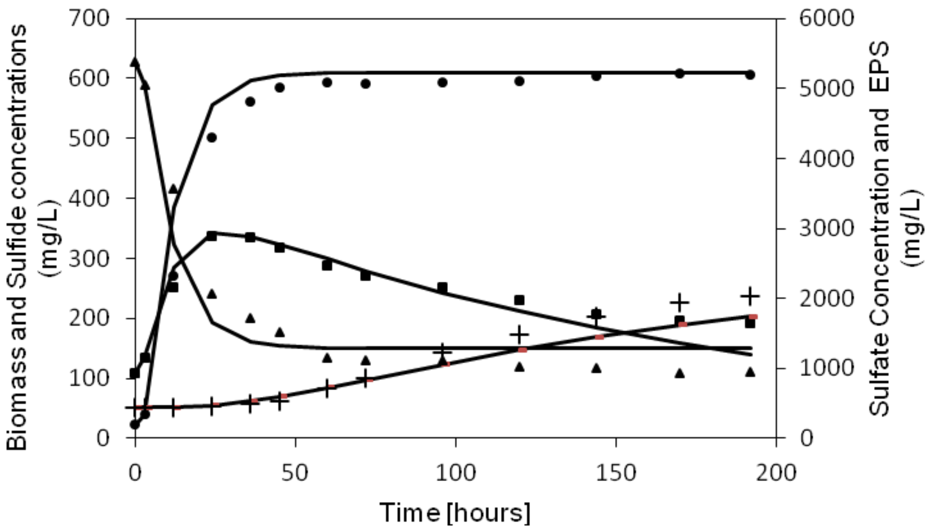

Fig. 1 Model validation, simulation data and experimental data
● – Sulfide, ■ – Biomass, ▲ – Sulfate, + – *EPS*

Estimation of the kinetic parameters

The growth kinetic parameters corresponding to Haldane-Boulton model were estimated by the rate of change of biomass production, using central finite differences according to the following equation:

$$\dot{x} \approx \frac{\Delta x}{\Delta t} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} \tag{2}$$

and a nonlinear multivariable regression for the rates of change of biomass production and experimental data (X, S, P and EPS) was done. POLYMATH 6.0 Professional software was used, the program allow applying effective numerical analysis techniques, and Liebenberg-Marquardt algorithm was using for this case. Table 1 contains the parameter's set obtained from the above methodology and Table 2 contains the structure of the kinetic rates and coefficient yield.

Table 1. Kinetic parameters for *Desulfovibrio alaskensis* 6SR

Model	μ_{max}, (h^{-1})	K_S, (mg/L)	K_P, (mg/L)	K_i	K_{EPS}, (mg/L)	n_{EPS}
Haldane-Boulton	39.84	86070	7.24	9850.24	----	----
Second order model for *EPS*	----	----	----	----	9.783E-07	2

Table 2. Kinetic rates and coefficient yields

Balance	**Equation**
Growth rate	$r_X = \mu X$
Dead rate	$r_d = \mu_d X$
Substrate coefficient yield	$Y_{S/X} = \dfrac{S_0 - S_i}{X_i - X_0}$
Product coefficient yield	$Y_{P/X} = \dfrac{P_i - P_0}{X_i - X_0}$

The mathematical model was simulated using the same software. A linear regression between the experimental data and the predicted data were obtained and overall correlation coefficient was calculated (see Table 3), as is showed in the Figs. 2 to 5:

Table 3. Correlation coefficients

Correlation coefficients r^2				**r^2 global**
Biomass	Sulfate	Sulfide	*EPS*	
0.9241	0.9479	0.9758	0.9737	0.9601

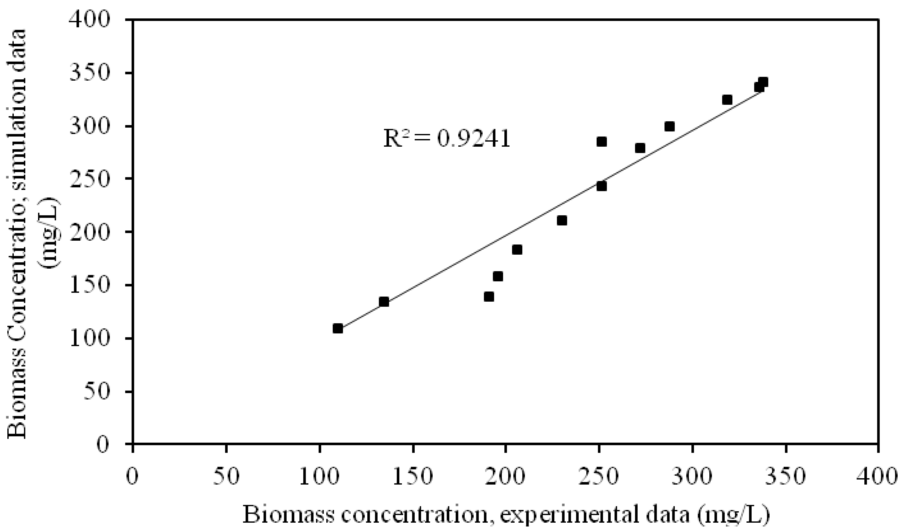

Fig. 2 Linear regression for biomass concentration data

Process modelling

The process models, as relationships of the input, output and inner variables, though incomplete and simplified, can be effective to describe the phenomena and the influences of great importance for control, optimization and better theoretical knowledge. The dynamic model concept plays a central role in automatic control. It is in fact on the basis of the time required for the development of the knowledge process that the total design, analysis and implementation of monitoring and control methods are carried out. Within the framework of bioprocesses, the most natural way to determine the models that will enable the characterization of the process dynamics is to consider the material balance (and possibly energy) of major components of the process.

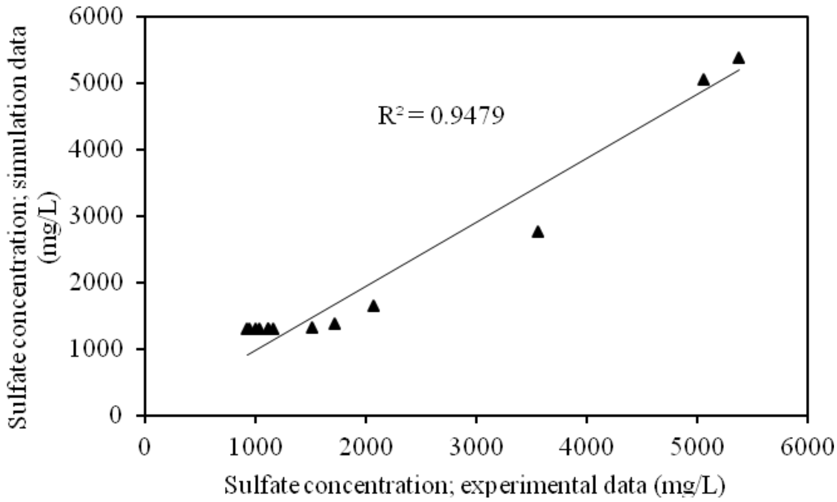

Fig. 3 Linear regression for sulfate concentration data

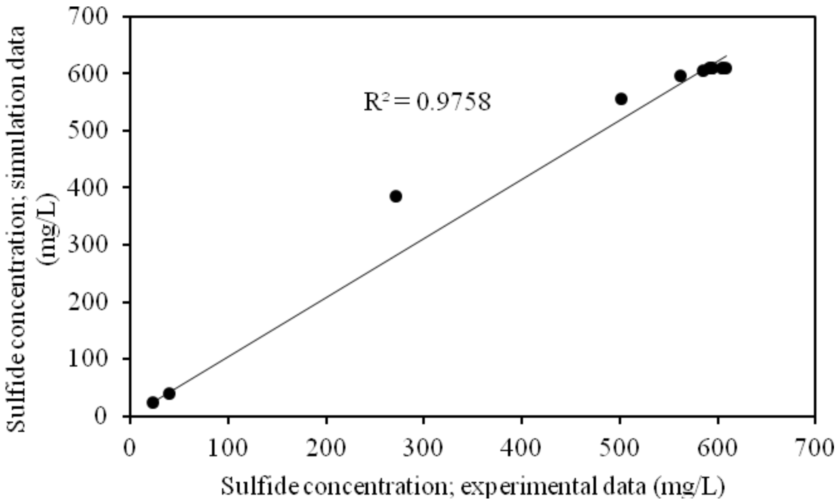

Fig. 4 Linear regression for sulfide concentration data

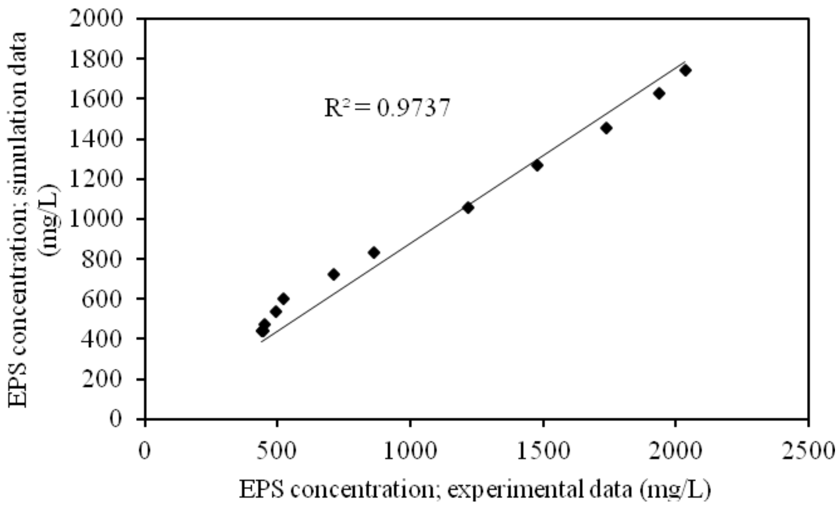

Fig. 5 Linear regression for *EPS* concentration data

One of the important aspects of the balance models is that they consist of two types of terms representing, respectively, conversion (i.e. kinetics substrates in terms of biomass and products) and the dynamics of transport (which regroups transit of matter within the process in solid, liquid or gaseous form and the transfer phenomena between phases). These models have various properties, which can prove to be interesting for the design of monitoring and control algorithms for bioprocesses [14].

From the above a classic mass balance equations considering non-structured kinetic model are considered to modeling the continuous sulfate-reducing bioreactor as follows:

Biomass (X) mass balance:

$$\frac{dX}{dt} = -DX + r_X - r_d \tag{3}$$

Sulfate (S) mass balance:

$$\frac{dS}{dt} = D\left(S_{in} - S\right) + \left(-Y_{S/X}\right)\left(r_X\right) \tag{4}$$

Sulfide (P) mass balance:

$$\frac{dP}{dt} = -DP + \left(Y_{P/X}\right)\left(r_X\right) \tag{5}$$

Extracellular Polymeric Substances (*EPS*) mass balance:

$$\frac{dEPS}{dt} = -DEPS + K_{EPS} X^{n_{EPS}} X_d \tag{6}$$

Dead biomass (X_d) mass balance:

$$\frac{dX_d}{dt} = -DX_d + \mu_d X \tag{7}$$

The following Figs. 6 and 7 correspond to the open-loop behavior of the bioreactor's concentration, note that the sulfate concentration reach a high level of 3115 mg/L at the corresponding steady-state and the *EPS* and dead biomass concentrations show an unstable behavior in continuous operation (see Fig. 7).

Proposed controller

The dynamic nonlinear model for a chemical reactor can be expressed as system (8), and the output measured for control purposes as (9):

$$\dot{x} = f(x) + g(x)u \tag{8}$$

$$y = h(x) = Cx \tag{9}$$

where $x = [S,\ X,\ P,\ EPS,\ X_D]^{\mathrm{T}} \in \Re_{+}^{5}$ is the corresponding state vector.

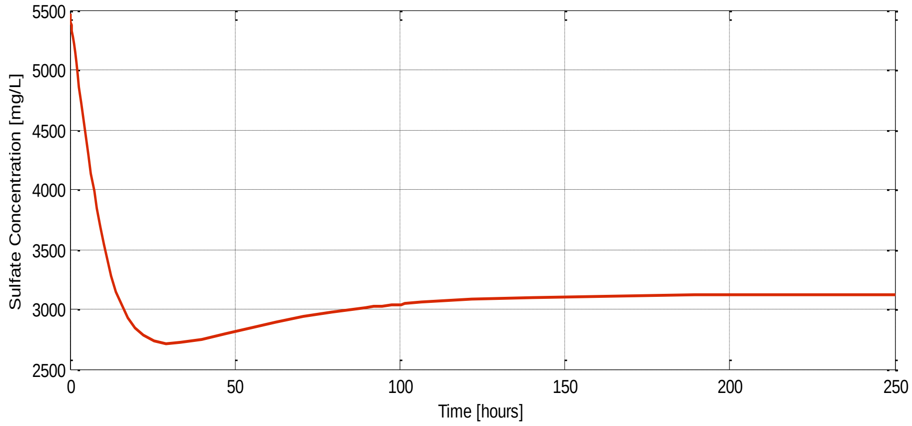

Fig. 6 Open-loop behavior of the control output for continuous bioreactor

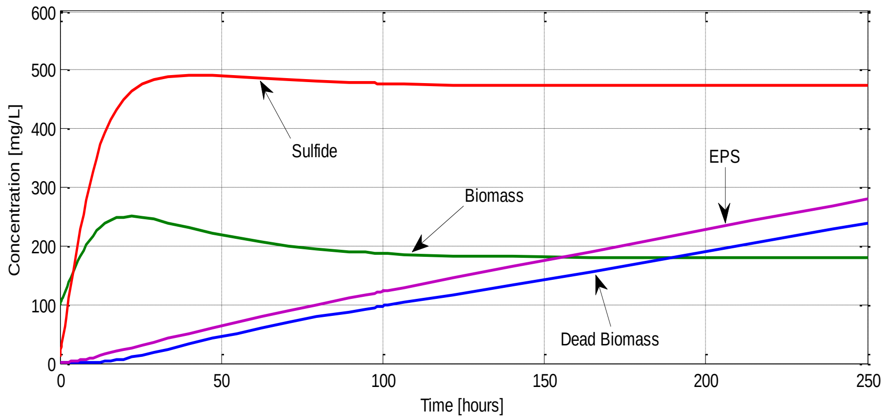

Fig. 7 Open-loop behavior of the uncontrolled state variables for continuous bioreactor

Now consider the set $\Phi \subset \Re^5$ as the corresponding physically realizable domain, such that:

$$\Phi = \begin{Bmatrix} (S, X, P, EPS, X_D) \in \Re_{+}^{5} \mid 0 \le S \le S_{in};\ 0 \le X \le X_{\max;};\ 0 \le P \le P_{\max}; \\ 0 \le EPS \le EPS_{\max};\ 0 \le X_D \le X_{D\max} \end{Bmatrix}$$

$f(x) \in C^{\infty}(x);\ \ f(0) = 0$ and $f(x) \le \Gamma\ \ \forall x \in \Re_{+}^{5}$, where $\Gamma < \infty$.

Now, consider the following assumptions:
***A*1**. For the realized control input vector $u(x(t))$, $\left(\|u(x(t))\| \le u_{\max}\right)$.
***A*2**. The matrix field $g(x)$ is bounded, i.e. for any $x \in \Re^n$, $\|g(x)\| \le g^+ < \infty$.

Now, in accordance to geometric-differential theoretical frame, the Lie derivative of the function $h(x)$ with respect to the vector field f, is denoted as $L_f h(x)$, where $L_f^r h$ is the *r*-order Lie derivatives and $dL_f^r h$ are the differentials of the r^{th}-order Lie derivatives defined recursively as follows:

$$
\begin{aligned}
&L_f^0 h := h\,,\ dL_f^0 h := dh = \left(\frac{\partial h}{\partial x_1},\cdots,\frac{\partial h}{\partial x_n}\right)\\
&L_f^1 h := \langle dh, f\rangle = \sum_{i=1}^{n}\frac{\partial h}{\partial x_i} f_i\,,\ dL_f^1 h := \left(\frac{\partial}{\partial x_1}\left(\sum_{i=1}^{n}\frac{\partial h}{\partial x_i} f_i\right),\ldots,\frac{\partial}{\partial x_n}\left(\sum_{i=1}^{n}\frac{\partial h}{\partial x_i} f_i\right)\right)\\
&L_f^r h := \langle dL_f^{r-1} h, f\rangle = L_f\left(L_f^{r-1} h\right),\quad r \ge 2
\end{aligned}
\tag{10}
$$

Definition (10) implies that there exists an invertible diffeomorphism, if the system is feedback linearizable, such that $(\vartheta,\xi) = \Phi(x)$ [10]. Therefore, the system (8) can be expressed as:

$$
\begin{aligned}
&\dot{\vartheta}_i = \vartheta_{i+1}\,;\ i = 1,\ 2,\ \ldots,\ r-1\\
&\dot{\vartheta}_r = \nu(\vartheta) + \psi(\vartheta) u\\
&\dot{\xi} = \gamma(\vartheta,\ \xi)\\
&y = \vartheta_1
\end{aligned}
\tag{11}
$$

Here: $\nu = L_f^r h(x)$; $\psi = L_g L_f^{r-1} h(x)$.

The sub-index *r* is defined as the relative degree of the system and it defines how many time derivatives of the measured output should be computed in order to obtain explicitly the control input.

Considering the following stable surface, with $j = e = \vartheta - \vartheta_{sp}$

$$
j = \vartheta_r - \vartheta_{r(0)} + \int \sum_{i=1}^{r} \beta_i \vartheta_i = 0 \tag{12}
$$

where *e* is the corresponding control error.

Note that the above surface provides stability to the system (11). Now, from (11) and (12), therefore it is possible to obtain (13):

$$\begin{aligned}
&\dot{\vartheta}_i = \vartheta_{i+1}\,;\; i = 1,\,2,\,\ldots,\,r-1\\
&\dot{\vartheta}_r = -\sum_{i=1}^{r}\beta_i\vartheta_i\\
&\dot{\xi} = \pi(\vartheta,\,\xi)\\
&y = \vartheta_1
\end{aligned} \tag{13}$$

In order to compress notation, system (13) can be represented as (14):

$$\begin{aligned}
&\dot{\vartheta} = A\vartheta\\
&\dot{\xi} = \pi(\vartheta,\,\xi)\\
&y = \vartheta_1
\end{aligned} \tag{14}$$

Here

$$A = \begin{bmatrix} 0 & 1 & \ldots & \ldots & 0\\ 0 & 0 & 1 & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ -\beta_1 & -\beta_2 & \ldots & \ldots & -\beta_r \end{bmatrix}$$

can be considered as a Hurwitz matrix with an adequate choosing of the design parameters β_i. The $\pi \in \Re^{n-r}$ system is the called inner or zero dynamics, which must be stable in order to provide detectability and stability for the system (14).

Ideal controller design

Now, proposing the following reaching dynamic as a desired trajectory:

$$\dot{j} = -c_1 j - \frac{c_2}{1+\exp(-|j|)} = 0 \tag{15}$$

For controller design, let us to consider the sliding-mode surface and the proposed reaching law, from Eqs. (12) and (15), therefore the following control law is obtained:

$$u = \psi(\vartheta)^{-1}\left(c_1 j - \frac{c_2}{1+\exp(-|j|)} - \nu(\vartheta) - \sum_{i=1}^{r}\beta_i\vartheta_i\right) \tag{16}$$

Substituting the above expression onto Eq. (11), produce the following closed-loop system:

$$\begin{aligned}
&\dot{\vartheta}_i = \vartheta_{i+1}\,;\; i = 1,\,2,\,\ldots,\,r-1\\
&\dot{\vartheta}_r = -c_1 j - \frac{c_2}{1+\exp(-|j|)} - \sum_{i=1}^{r+1}\beta_i\vartheta\\
&\dot{\xi} = \pi(\vartheta,\xi)\\
&y = \vartheta_1
\end{aligned} \tag{17}$$

Now, analyzing the closed-loop stability of the sliding surface, let us to propose the following positive quadratic function:

$$V = \frac{j^2}{2} \tag{18}$$

Considering its time derivative and substituting Eq. (15):

$$\dot{V} = j\,\dot{j} \tag{19}$$

Applying the Lyapunov stability criteria to Eq. (19):

$$j\left(-c_1 j - \frac{c_2}{1+\exp\left(-|j|\right)}\right) < 0$$

Therefore, the equivalent algebraic forms:

$$-c_1 j - \frac{c_2}{1+\exp\left(-|j|\right)} < 0 \tag{20}$$

which is negative if: $c_1 > 0$ and $c_2 > 0$ ♦

Note that for our case, the relative degree is $r = 1$ and the state vector is defined as:

$$x = \left[X,\, S,\, P,\, EPS,\, X_d\right]^{\mathrm{T}}$$

The nonlinear vectors are defined by:

$$f(x) = \begin{bmatrix} r_X - r_d \\ (-Y_{S/X})(r_X) \\ (Y_{P/X})(r_X) \\ K_{EPS} X^{n_{EPS}} X_d \\ \mu_d X \end{bmatrix},\; g(x) = \begin{bmatrix} -X \\ (S_{in} - S) \\ -P \\ -EPS \\ -X_d \end{bmatrix},\, u = D$$

While, the transformed space employing Lie derivatives is:

$$\dot{\vartheta} = \dot{j} = D\left(S_{in} - S\right) + \left(-Y_{S/X}\right) r_x \tag{21}$$

The inner dynamics is:

$$\dot{\pi} = \begin{bmatrix} -DX + r_X - r_d \\ -DP + (Y_{P/X})(r_X) \\ -DEPS + K_{EPS} X^{n_{EPS}} X_d \\ -DX_d + \mu_d X \end{bmatrix}$$

The proposed control input in the original space state representation is:

$$u = D = \frac{1}{\left(S_{in} - S\right)}\left(c_1 j - \frac{c_2}{1+\exp\left(-|j|\right)} - S + \left(-Y_{S/X}\right) r_x\right) \tag{23}$$

The measured system output is:

$$y = S \tag{24}$$

No ideal controller

Because the difficulty to obtain accurate kinetic modeling the ideal controller given by Eq. (23), could not be realizable, therefore an estimation of this important term must be provided, therefore the named non ideal controller is expressed as:

$$u = D = \frac{1}{\left(S_{in} - \hat{S}\right)}\left(c_1 j - \frac{c_2}{1+\exp\left(-|j|\right)} - \hat{S} + \left(-Y_{S/X}\right) \hat{r}_x\right)$$

where $\left(-Y_{S/X}\right)\hat{r}_x$ is an estimation of $\left(-Y_{S/X}\right) r_x$ and $\hat{S}$ is the estimate sulfate concentration.

Now, from the system (17), let us to considerd the controlable subspace, where now it is assumed that the term ν is uncertain and it is considered as an extended state variable with unknown dynamics, from the above the following state observer is considered:

$$\begin{aligned} \dot{\hat{\vartheta}} &= \hat{\nu} + \psi\left(\hat{\vartheta}\right)u + k_1\left(y - \hat{y}\right) \\ \dot{\hat{\nu}} &= k_2\left(y - \hat{y}\right) \\ \dot{K} &= -\beta\, abs\left(y - \hat{y}\right)^{1/m} \end{aligned} \tag{25}$$

By defining $\hat{\aleph} = \begin{bmatrix} \hat{\vartheta} \\ \hat{\nu} \end{bmatrix}$, $\hat{\Im} = \begin{bmatrix} \hat{\vartheta} + \Psi\left(\hat{\vartheta}\right)u \\ 0 \end{bmatrix}$ and $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$, Eq. (15) can be rewritten as:

$$\begin{aligned} \dot{\hat{\aleph}} &= \hat{\Im} + K\left(y - \hat{y}\right) \\ \dot{K} &= -\beta\, abs\left(y - \hat{y}\right)^{1/m} \end{aligned} \tag{26}$$

Here the dynamic equation for K is an adaptation algorithm that updates the time-varing control gain and β is a parameter design.

In order to prove the convergence of the proposed observer, lets consider the dynamic equation of the estimation errors, $\varepsilon = \aleph - \hat{\aleph}$, as follows:

$$\begin{aligned} \dot{\varepsilon} &= \Im - \hat{\Im} + K\varepsilon \\ \dot{K} &= -\beta\left|\varepsilon\right|^{1/m} \end{aligned} \tag{27}$$

Because the error is a finite quantity, there should be a constant L that:

A3. $\left|\Im - \hat{\Im}\right| \le L\left|\aleph - \hat{\aleph}\right|$

Taking norms to both sides of Eq. (27) and applying ***A*3** it is obtained:

$$|\dot{\varepsilon}| \le L|\varepsilon| + K|\varepsilon| \tag{28}$$

Now, to solve the system given by Eq. (27), consider the function $|\varepsilon|$ as a positive continuous function on the integration interval $[\alpha, \beta]$; if Λ is the maximum of the function for the observer's gain dynamic on the domain $[\alpha, \beta]$, then $abs(\varepsilon)$ is bounded, i.e. $abs(\varepsilon) \le \Lambda \quad \forall\, t \in [\alpha, \beta]$, hence:

$$|\varepsilon|^{1/n} \le \Lambda^{1/n} \quad n > 0 \Rightarrow \int_{\alpha}^{\beta} |\varepsilon|^{1/n} \le \Lambda^{1/n}(\beta - \alpha) \tag{29}$$

Here, n is restricted to be an odd number, i.e. $n = 2p + 1, p \in Z^{+}$. Therefore, for p large enough, the following limit is obtained:

$$\lim\sup \int_{\alpha}^{\beta} |\varepsilon|^{1/(2p+1)} \le \lim\sup \Lambda^{1/(2p+1)}(\beta - \alpha) \le (\beta - \alpha) \tag{30}$$

Applying the equality $|\varepsilon| = sign(\varepsilon)\varepsilon$ to Eq. (28), another quota can be found:

$$sign(\dot{\varepsilon})\dot{\varepsilon} \le \big(L - \beta(b - a)\big) sign(\varepsilon)\varepsilon \tag{31}$$

By solving Eq. (31) it is possible to note that the error is bounded by:

$$\varepsilon \le \varepsilon_0 \exp\Big(sign(\dot{\varepsilon})^{-1} sign(\varepsilon)\big(L - \delta(\beta - \alpha)\big)t \Big) \tag{32}$$

Therefore the estimation error will be asymptotically and exponentially stable if:

$$\delta > (\beta - \alpha)^{-1} L \tag{33}$$

The form of the considered observer in the original state space is given by:

$$\frac{d\hat{S}}{dt} = D\left(S_{in} - \hat{S}\right) + \left(-Y_{S/X}\right)\left(\hat{r}_X\right) + k_1\left(S - \hat{S}\right) \tag{34}$$

$$\dot{\hat{r}}_X = k_2\left(S - \hat{S}\right) \tag{35}$$

where

$$\dot{K} = -\beta\, abs\left(S - \hat{S}\right)^{1/m} \quad \text{with } K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}^{\mathrm{T}} \tag{36}$$

From the above, can be concluded that given the asymptotically and exponentially convergence properties of the uncertainty estimator, the non ideal controller recovers the properties of the named ideal controller.

Results and discussion

In this section a set of numerical simulations was done in order to provide the adequate performance of the proposed control methodology. It is considered the following initial values for the bioreactor concentration which was taken in account on the corresponding simulations: X_0 = 134.73 mg/L; S_0 = 5057.47 mg/L; P_0 = 35 mg/L, EPS_0 = 0.0 mg/L and finally X_{d0} = 0.0 mg/L. The considered control input is the dilution rate (properly, the input flow) and the controlled and measured variable is the sulfate concentration (control output), it is considered that the kinetic term related with the sulfate consumption is assumed unknown. The controller's parameter are c_1 = 1 h^{-1} and c_2 = 1. To simulate the closed-loop operation the proposed controller is turned up at 50 hours, where the considered set point is S_{sp} = 2000 mg/L of sulfate concentration, after that, at 100 hours a step change on the reference concentration is considered as S_{sp} = 1500 mg/L of sulfate concentration. For comparison purposes a standard PI controller, tuned under the IMC tuning guidelines [17], which are widely employed in industrial practice, with a proportional gain of K_P = 0.00272 h^{-1} and integral time of τ = 40 hours and a well tuned SM Controller (SM) [12] with gain of K_{SM} = –0.1 h^{-1} is implemented too. The proposed observer (Eqs. (34-36)) is tuned with the following set of parameters; k_1 = 25 h^{-1}, k_2 = 10 h^{-1}, β = 1 and m = 0.3. The initial conditions for the observer are $\hat{S}_0$ = 5250 mg/L and K_0 = 1 h^{-1}.

Fig. 8 is related with the closed-loop behavior of the sulfate concentration, when the controllers are turned the SM and the proposed controllers act and lead the corresponding concentration to the required set point (2000 mg/L) in a smooth form, the SM controller has an approximate settling time of two hours and the proposed one acts almost immediately, without overshoots, the linear PI controller has an important overshoot of around 2500 mg/L and a very slow response, moreover it is not able to reach the required sulfate concentration (2000 mg/L); after that when the set point is changed to the new reference (1500 mg/L) the standard PI controller shows again a poor performance reaching the proposed closed-loop steady-state with a large settling time of 140 hours approximately, with an overshoot of 500 mg/L in the sulfate concentration; on other hand the SM controller acts satisfactorily, however an small off-set is generated too, finally the proposed methodology reach the corresponding set point without off-set, settling time an overshoot. Now, the closed-loop behavior of the uncontrolled states (inner or zero dynamics) is showed in Fig. 9, can be observed an unstable behavior of the Sulfide, *EPS* and dead biomass concentrations, predicted by the bioreactor´s model, the only stable concentration correspond to the biomass concentration, from this situation, this system can considered as a minimum phase, which is an important issue from the process operation and control. Fig. 10 shows the performance index of the process under the action of the three considered controllers, it is proposed he ITSE (integral time square error) [15] as performance index, as mentioned above, when the controller is turned at 50 hours with a set point of 2000 mg/L of sulfate concentrations for the SM and the proposed controllers show similar performance index, however, the proposed methodology has the better performance; the linear PI controller, for the above mentioned produce the largest ITSE. Further simulations where realized, considering noisy measurements in the sulfate concentration, the noise effect was modeled as a sinoidal function (45 $Sin(10t)$); under this framework the Fig. 11 shows the closed-loop performance of the proposed observer for the sulfate concentration, can be noticed that the observer is able to filter the noisy measured signal adequately. Fig. 12 shows the performance of the control effort, related with the input flow (dilution rate), under noisy measurements, the nominal value of the dilution rate in 0.025 h^{-1}, at 50 hours of the process start, the controller is turned-on; the dilution rate is diminished to around 0.01 h^{-1} in order to lead to the sulfate concentration to the required set point of 2000 mg/L at 100 hours the set point is changed to

1500 mg/L of the sulfate concentration and the controller' response decrease the dilution rate to 0.005 h^{-1} as can be observed, the controller is on a reachable operation flow range (from 0 h^{-1} to 0.01 h^{-1}) which is under the nominal dilution rate value (0.025 h^{-1}), the controller close the flow to zero in order to increase the residence time in order to allow that the sulfate concentration can be diminish; note that the noisy measurements affect the control input induce oscillations, however these oscillations are on a rank of 0.0025 h^{-1}, and can be considered small enough to not affect the controller' performance. Finally, Fig. 13 contains the dynamic performance of the uncertainty observer, where is observed a satisfactory behavior due the satisfactory convergence (around 40 hours) of the estimate uncertain term to the named real uncertainty, despite of the noisy measurements of the sulfate concentration, considering that the bioreactor is in closed-loop operation at 50 hours, the global performance of the uncertainty observer based controller is considered adequate.

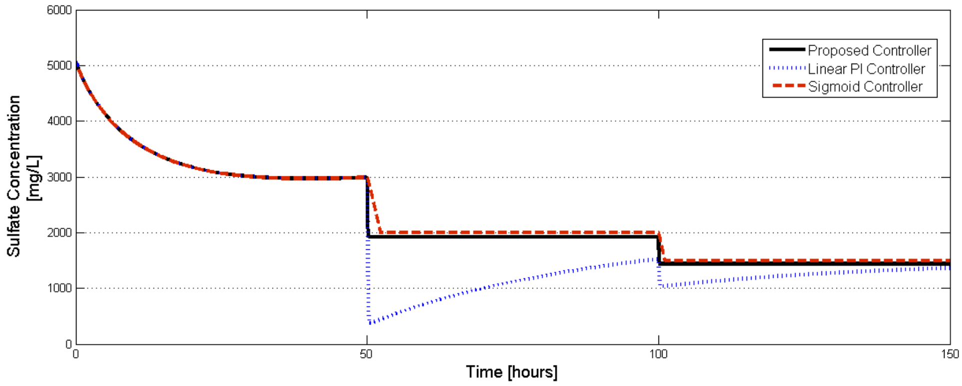

Fig. 8 Closed-loop behavior of the control output

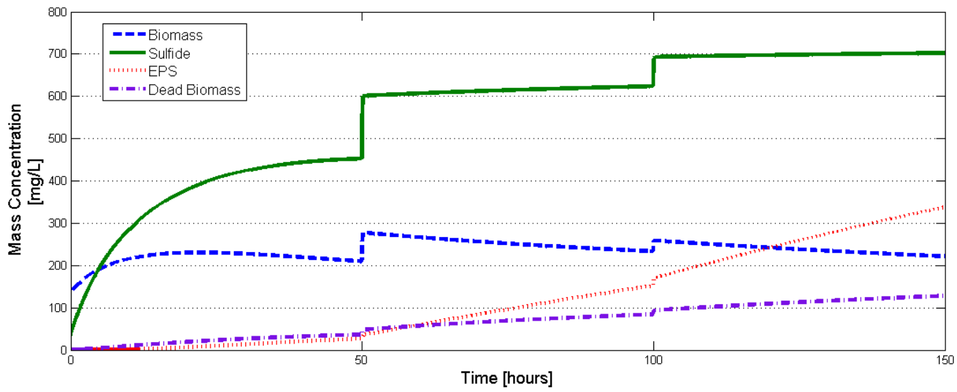

Fig. 9 Closed-loop behavior of the uncontrolled state variables

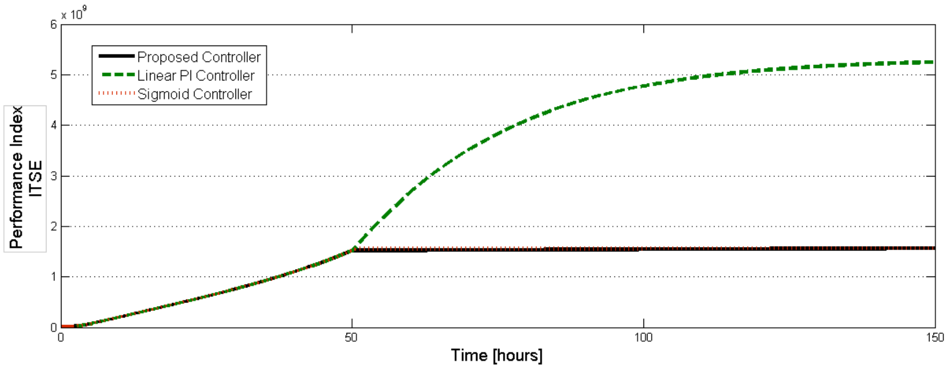

Fig. 10 Dynamic behavior of the performance index (ITSE)

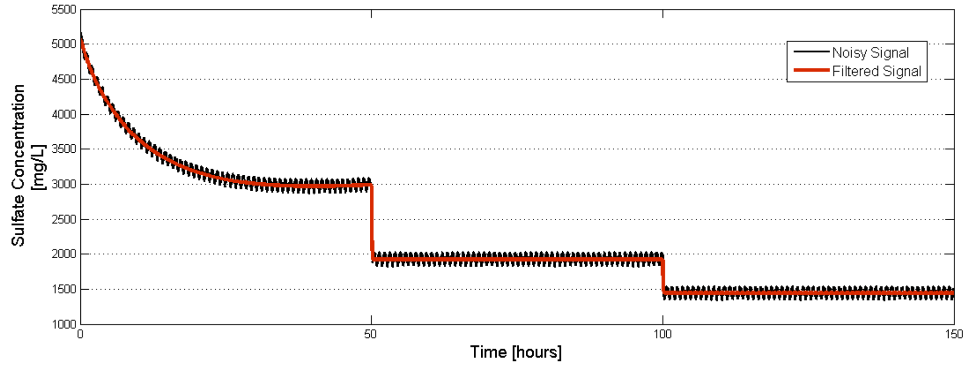

Fig. 11 Closed-loop behavior of the sulfate concentration under noisy measurements

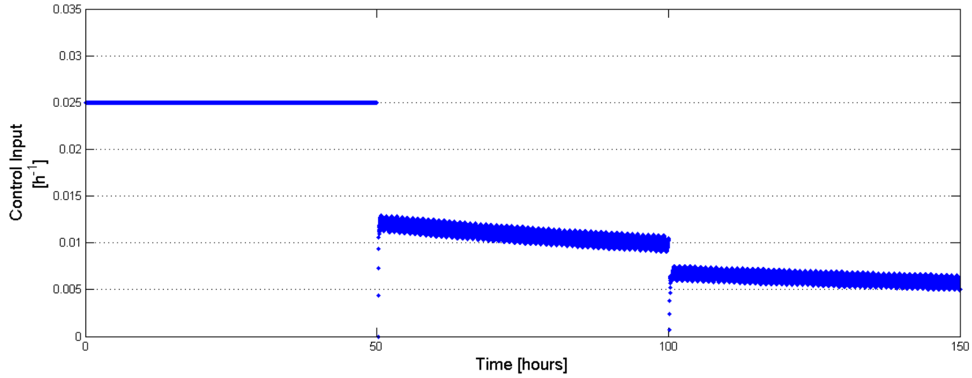

Fig. 12 Control effort, under noisy measurements

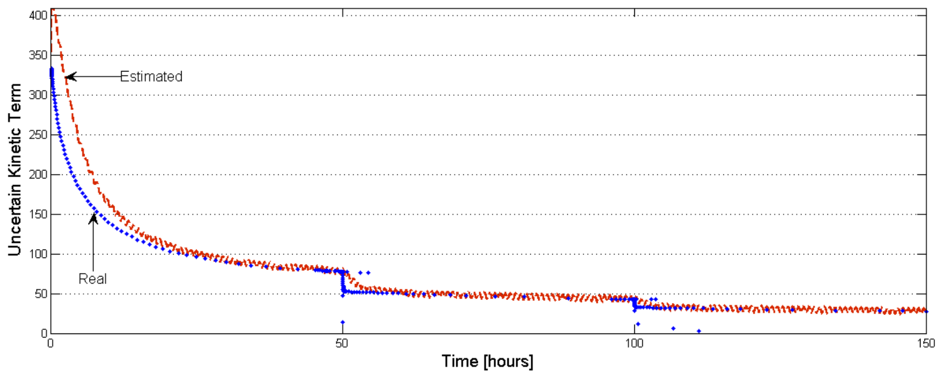

Fig. 13 Dynamic performance of the uncertainty observer, under noisy measurements

Conclusion

In this paper is presented a kinetic model for the cell growth of a sulfate-reducing bacterium *Desulfovibrio alaskensis* which has good agreement with experimental data, in accordance with the corresponding correlation coefficient. This kinetic model is extended to continuous stirred bioreactor model. A class of smooth controller to regulated the sulfate concentration via the dilution rate as control input is proposed, in order to show better performance in comparison with other controllers, the proposed methodology avoids the named chattering behavior, which is an undesired phenomena present on typical sliding-mode controllers. Numerical simulations allow concluding an unstable behavior of the inner dynamic of the bioreactor.

References

1. Alford J. S. (2006). Bioprocess Control: Advances and Challenges, Computers and Chemical Engineering, 30, 1464-1475.
2. Bastin G., D. Dochain (1990). On-line Estimation and Adaptive Control of Bioreactors, Process Measurement and Control, Vol. 1, Elsevier, Amsterdam.
3. Betancourt M. J., J. A. Moreno, I. Moreno-Andrade, G. Buitrón (2006). Practical Optimal Control of Fed-batch Bioreactors for the Wastewater Treatment, International Journal of Robust and Nonlinear Control, 3, 173-190.
4. Chidambaram M., R. P. A. See (2002). Simple Method of Tuning PID Controllers for Integrator/Dead-time Processes, Computer & Chemical Engineering, 27, 211-215.
5. Cord-Ruwisch R. (1985). A Quick Method for Determination of Dissolved and Precipitated Sulfides in Cultures of Sulfate-reducing Bacteria, J Microbiol Methods, 4, 33-36.
6. Davidson P., F. Wernstedt (2002). Software Agents for Bioprocess Monitoring and Control, Journal of Chemical Technology & Biotechnology, 77(7), 761-766.
7. Feio M. J., V. Zinkevich, I. B. Beech, E. L. Brossa, P. Eaton, J. Schmitt, J. Guezennec (2004). *Desulfovibrio alaskensis* sp. nov., a Sulphate Reducing Bacterium from a Soured Oil Reservoir, International Journal of Systematic and Evolutionary Microbiology, 54(Pt5), 1747-1752.
8. Harms P., Y. Kostov, G. Rao (2002). Bioprocess Monitoring, Current Opinion in Biotechnology, 13(2), 124-127.
9. Hungate E. (1969). A Roll Tube Method for Cultivation of Strict Anaerobes, In: Methods in Microbiology, Norris J. R., D. W. Ribbons (Eds.), London, Academic Press, Vol. 3B, 117-132.
10. Isidori A. (1995). Nonlinear Control Systems, Springer-Verlag, 3rd Edition.
11. López-Pérez P. A., R. Aguilar-López (2009). Dynamic Nonlinear Feedback for Temperature Control of Continuous Stirred Reactor with Complex Behavior, Journal of Applied Research and Technology, 7(2), 202-217.
12. López-Pérez P. A., M. I. Neria-González, R. Aguilar-López (2013). Nonlinear Control Design with Application to a Class of Continuous Bioreactor, Theoretical Foundations of Chemical Engineering, 47(5), 566-572.
13. Neria-Gonzáalez M. I., J. C. Figueroa-Estrada, M. R. Cruz-Diaz, R. Aguilar-López (2011). Adaptive Smooth Observer Design for State Estimation in *Desulfovibrio alaskensis* 6SR Cultures, Revista Mexicana de Ingeniería Química, 10(1), 137-146.
14. Neria-González I., E. T. Wang, F. Ramírez, J. M. Romero, C. Hernández-Rodríguez (2006). Characterization of Bacterial Community Associated to Biofilms of Corroded Oil Pipelines from the Southeast of Mexico, Anaerobe, 12, 122-133.
15. Ogunnaike B. A., W. H. Ray (1994). Process Dynamics, Modeling, and Control, Oxford University Press, USA.
16. Posgate J. R. (1981). Sulfate-reducing Bacteria, Cambridge University Press, New York, USA.
17. Rivera E. D., M. Morari, S. Skogestad (1986). Internal Model Control 4. PID Controller Design, Industrial & Engineering Chemistry Process Design and Development, 1, 252-265.
18. Spear M. (2005). Time to Turn Up the Speed? Process Control Developments Continue to Outrun their Uptake by Industry, Chemical Processing, 23, 5, 345-349.
19. Vojinovic V., J. M. S. Cabral, L. P. Fonseca (2006). Real-time Bioprocess Monitoring. Part I: In situ Sensors, Sensors and Actuators B, 114, 1083-1091.

20. Wikström K. Å. P., K. Hallberg (2000). A Fast and Simple Turbidimetric Method for the Determination of Sulfate in Sulfate-reducing Bacterial Cultures, Microbiol Meth, 41, 179-184.
21. Ziegler J. G., N. B. Nichols (1942). Optimum Settings for Automatic Controllers, Trans. ASME, 64, 759-765.

Prof. Maria Isabel Neria-Gonzalez, Ph.D.
E-mail: ibineria@hotmail.com

Maria Isabel Neria-Gonzalez is a biochemical engineer from Universidad Autónoma Metropolitana (1999). She ears M.Sc. and Ph.D. degrees in microbiology science from Escuela Nacional de Ciencias Biológicas, IPN (2002 and 2006, respectively). She was a postdoctoral fellow in the Departamento de Biotecnología y Bioingeniería at CINVESTAV-IPN and in the Microbiology Department of Provence University, France. She is an author and/or coauthor of 22 published papers in indexed journals. Currently, she is a full professor at Tecnológico de Estudios Superiores de Ecatepec.

Ricardo Aguilar-Lopez, Ph.D.
E-mail: raguilar@cinvestav.mxmailto:

Ricardo Aguilar-Lopez received the B.Sc., M.Sc., and Ph.D. degrees from the Universidad Autónoma Métropolitana, Mexico, all of them in chemical engineering. Besides, he received a Ph.D. in automatic control from the Centro de Investigación y de Estudios Avanzados (CINVESTAV). Currently, he has been with the Department of Biotechnology and Bioengineering at the CINVESTAV. His research interests include modeling, dynamic analysis and control applications in chemical and biochemical systems, nonlinear observer design, and chaos engineering applications. He has published around 122 technical papers in different international journals and 65 contributions in chapters of specialized control books and conferences proceedings.