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On an Intuitionistic Fuzzy Approach for Decision Making in Medicine: Part 1

Ludmila Todorova[*], Krassimir Atanassov, Stefan Hadjitodorov, Peter Vassilev

Centre of Biomedical Engineering Prof. Ivan Daskalov – Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., Sofia 1113, Bulgaria
E-mail: {lpt, sthadj, ilywrin}@clbme.bas.bg, krat@bas.bg

[*] *Corresponding author*

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Abstract: *Determining the readiness of the patient for weaning from long-term mechanical ventilation is one of the main tasks that need to be overcome by clinician in the intensive care units. In the present paper it is defined as a pattern recognition problem and four classification methods are applied in order to solve it: Stepwise discriminant analysis (SDA), stepwise logistic regression (SLR); intuitionistic fuzzy Voronoi diagrams and nonpulmonary weaning index (NPWI), applied to 17 features (variables). The result of each method is represented by relative recognition accuracy. As a final estimate of the classification a value which is received as an aggregated result from the four estimates received by each of the procedures.*

Keywords: *Intuitionistic fuzzy sets, Pattern recognition, Mechanical ventilation, Weaning, Readiness to weaning.*

Introduction

In recent decades the mechanical ventilation has established itself as primary life-saving method in the intensive care units. The percentage of the patients, who receive ventilatory support in circumstance of critical care, varies between 20 and 60%. Clinical practice, however, shows that this lifesaving method is one of the main reasons for the development of lethal nosocomial infections and septic complications. In such patients the risk is several times higher, as some authors report, and it grows in proportion of the duration of breathing support. It is reported that for cumulative frequency of pneumonias from 8.5% during the first 3 days after beginning of ventilation, 21.1% during the first 7 days, 32.4% to the 14th day and 45.6% in cases with mechanical ventilation with duration over 14 days [3, 4, 10, 13, 19]. The accrued clinical experience and research form one of the main tasks – making a prognosis for the appropriate moment for the beginning of weaning from long-term (at least 7 days) mechanical ventilation.

Intensive therapy and examination requires interdisciplinary approach and the refinement of the indices used to determine the moment of weaning from mechanical ventilation. The application of different mathematical models for determination of the method that would be used to carry out the transition to spontaneous breathing is a very promising approach that has found application in various publications [6, 8, 9].

One of the special features of the data obtained by medical examinations is that a particular disease affects different patients with varying strength. This may be represented with different degrees of membership of the patient to the particular class, corresponding to the particular disease. Because of that when processing medical information the apparatus of fuzzy sets, in particular – of intuitionistic fuzzy sets is being employed more frequently. [2, 5, 7, 12, 14] since they provide an adequate description of the objects considered and permit pattern recognition for non-strict membership of the images.

The question arises in the setting of decision making as follows. Given a single item, a number of experts are asked about the membership of the item in a certain class, i.e. whether it has a particular property. The answers may not be definite "yes" or "no", but values between 0 and 1, which we interpret as degrees of membership or probabilities. The task is to aggregate these values into a single value, that we allow to be Intuitionistic Fuzzy (IF). Indeed, given that the initial fuzzy values may not be the same, it is very natural to allow some degree of indeterminacy in the aggregated result. An IF value consists of two real numbers μ and ν in the interval [0, 1] satisfying $\mu+\nu\le 1$. We think of μ and ν as degrees of membership and non-membership, respectively, and we think of the number $\pi = 1-\mu-\nu$ as the degree of indeterminacy. Clearly, in the fuzzy case we have $\mu+\nu=1$, so that the degree of indeterminacy π is 0. A detailed account of IF sets and related concepts can be found in [1].

Algorithm for aggregation of intuitionistic fuzzy estimates

Oftentimes one pattern recognition problem is solved by different classification procedures. In this case we need to solve the matter with aggregating the results obtained. When the results are intuitionistic fuzzy estimates for the degrees of membership, non-membership and indeterminacy the matter of reducing the indeterminacy also arises. In [18] are proposed two algorithms for aggregating the values, obtained through the use of two or more classification procedures when solving a particular pattern recognition problem. They are applicable to any number of classification procedures, used in a given pattern recognition problem, i.e. – for making a decision about the membership of these images to a certain class. As a result of the application of each of the proposed algorithms we obtain a generalized IFS- estimate [1] of the classification. The first algorithm is described below:

The problem statement is as follows. We are given m fuzzy (or probabilistic) values, which we denote by $\sigma_1,\sigma_2,...,\sigma_m$. The task is to produce an IF value whose degrees of membership, non-membership and indeterminacy we denote by μ, ν and π respectively.

Let us denote

$$\sigma_{\min} = \min\{\sigma_1,\sigma_2,...,\sigma_m\} \tag{1}$$

$$\sigma_{\max} = \max\{\sigma_1,\sigma_2,...,\sigma_m\} \tag{2}$$

We first consider the basis cases for our algorithms, $m=2$ and $m=3$.

1. if $m=2$, we let

$$\mu = \sigma_{\min} \tag{3}$$

$$\nu = 1-\sigma_{\max} \tag{4}$$

$$\pi = \sigma_{\max}-\sigma_{\min} \tag{5}$$

2. if $m = 3$, let us denote the mid-value by σ, i.e. the three fuzzy values are $\sigma_{\min} \le \sigma \le \sigma_{\max}$.

Let us denote by σ' the "conjugate" value of σ, i.e. the symmetric one with respect to the mid-point of the segment $\sigma_{\min}\sigma_{\max}$,

$$\sigma' = \sigma_{\min} + \sigma_{\max} - \sigma \quad (6)$$

We then let

$$\mu = \min\{\sigma, \sigma'\} \quad (7)$$

$$\nu = 1 - \max\{\sigma, \sigma'\} \quad (8)$$

$$\pi = |\sigma_{\min} + \sigma_{\max} - 2\sigma| \quad (9)$$

3. for $m > 3$.

The algorithm is a very simple one. We take the values around the median of the sequence $\sigma_1, \sigma_2, ..., \sigma_m$, that is we take the two values in the middle if m is even, and the median plus the two values around it if m is odd. We then apply the relevant basis case to the 2 or 3 values, respectively.

Let us have a set of images, and for each of them the pattern recognition problem is solved by no more than k in number classification methods. Let the result of the classification from each of the methods is a number in the interval [0, 1]. We shall denote by σ_s the value of the estimate of the $s-$th method for a particular image ($s = 1,2,...,k$). Then we have k in number fuzzy estimates $\sigma_1, \sigma_2, ..., \sigma_k$, whose values we input in the interval [0, 1]. It is possible for some of these estimates to coincide. The task that we set before us is to produce a common estimate based on the estimates of the values generated by the individual methods, and that would also reduce the degree of indeterminacy.

The proposed algorithm is applied to the solution of the classification problem for making a prognosis of the appropriate moment for the beginning of weaning from long-term mechanical ventilation. Retrospectively 151 patients were examined, treated and discharged from Respiratory center of the SUH – Aleksandrovska for a period of eleven years (1989-1999). All of them were put under long-term mechanical ventilation for more than 7 days (26.12±11.09). In order to solve the particular pattern recognition problem each patient is represented by two n-dimensional vectors (in our case, n = 17), i.e.:

$$x = (x_1, x_2, ..., x_n) \quad (10)$$

where: x_1 – fever; x_2 – hemoglobine; x_3 – hematocrit; x_4 – Leuc; x_5 – RUE; x_6 – total blood protein (tbp); x_7 – blood albumin (alb); x_8 – blood sugar (bs); x_9 – lactate; x_{10} – fraction of inspired oxygen FiO_2; x_{11} – arterial oxygen partial pressure PaO_2; x_{12} – arterial carbon dioxide tension $PaCO_2$; x_{13} – ratio PaO_2 /FiO_2; x_{14} – heart rate (Ps); x_{15} – systolic arterial pressure (RRs); x_{16} – diastolic arterial pressure (RRd); x_{17} – mean arterial pressure (RRm.). One of the vectors is composed of the values of the respective features in day before the beginning of weaning. In this case we agree to assume that the patient belongs to class ω_1 – "not ready for weaning". The second vector consists of the values of the features in the day the weaning begins, i.e. the values on the basis of which the treating doctor has evaluated the patient as “ready for weaning”. In this case we agree to assume that the patient belongs to class ω_2 – “ready for weaning”.

We apply four classification methods: Stepwise discriminant analysis (SDA), stepwise logistic regression (SLR) [11, 20]; “intuitionistic fuzzy” Voronoi diagrams (IFVD) [15, 16] and nonpulmonary weaning index (NPWI) [17].

All of them determine the membership of a patient to one of the two considered classes. The results are represented by a fuzzy estimate. This estimate is obtained as follows:

- For the method using SDA.

The stepwise discriminant analysis calculates two linear discriminant functions as follows:

$$F'(x)=\sum_{i=1}^{n}\varpi_i' x_i + a' \tag{11}$$

for the class ω_1;

$$F''(x)=\sum_{i=1}^{n}\varpi_i'' x_i + a'' \tag{12}$$

for the class ω_2,

where ϖ_i', ϖ_i'' are discriminant coefficients and a', a'' are constants. It is assumed that the probability distribution is normal and the classes’ covariance matrices are equal.

$F'(x)$ and $F''(x)$ define the class membership of each patient with vector *X*. If the first function $F'(x)$ is greater than $F''(x)$ then *X* belongs to class ω_1, else – to class ω_2.

Let for a given patient the values of the two linearly discriminant functions, which are calculated with the stepwise discriminant analysis, be respectively:

$$F_i'=\sum_{j=1}^{17} w_i' x_{ij} + a' \tag{13}$$

$$F_i''=\sum_{j=1}^{17} w_i'' x_{ij} + a'' \tag{14}$$

where w_i', w_i'', a', a'' are discriminant coefficients and a constant coefficient, and x_{ij} is the value of the $j-$th index for the $i-$th patient.

Let

$$sign(x)=\begin{cases}1, & x \geq 0\\ 0, & x < 0\end{cases} \tag{15}$$

and

$$p_i = F_i'' - F_i' \tag{16}$$

$$p_{\max} = \max(p_i) \tag{17}$$

$$p_{\min} = \min(p_i) \tag{18}$$

where $i = \overline{1,m}$, then

$$\sigma_i = \tfrac{1}{2}(1-(1-sign(p_i))\frac{p_i}{p_{\min}} + sign(p_i)\frac{p_i}{p_{\max}}),\ 0 \le \sigma_i \le 1 \quad (19)$$

i.e.

$$\sigma_i = \begin{cases} \dfrac{p_i}{2}\left(\dfrac{1}{p_{min}} + \dfrac{1}{p_{max}}\right), p_i \ge 0 \\ 0,\ p_i < 0 \end{cases} \quad (20)$$

- For the method using stepwise logistic regression.

The assumption in stepwise logistic regression is that the difference between the logarithms of the conditional *pdf* of the two classes is linear with respect to $x_1, x_2, ..., x_n$ i.e.:

$$\frac{p(\omega_1 / x)}{p(\omega_2 / x)} = \exp(\beta'_0 + \beta^T x) \quad (21)$$

Discrimination between the two classes depends on the ratio $\frac{p(\omega_1 / x)}{p(\omega_2 / x)}$, whereby:

x is put into ω_1, if

$$\frac{p(\omega_1 / x)}{p(\omega_2 / x)} > 1 \quad (22)$$

x is put into ω_2, if

$$\frac{p(\omega_1 / x)}{p(\omega_2 / x)} < 1 \quad (23)$$

In stepwise logistic regression, independent (predictive) variables are included or eliminated in a stepwise manner. In our investigation stepwise forward regression is used, whereby new variables are added until the model reaches optimal performance. Similar stepwise procedure is used in stepwise discriminant analysis also.

To make a decision for the logistic regression we compare the values of the linear function

$$F_{LR} = \beta_0 + \beta^T x \quad (24)$$

with threshold value $F_{\lim}$.

Let

$$f_i = F_{\lim} - F'_{LR}(x_i) \quad (25)$$

$$f_{\max} = \max(f_i) \quad (26)$$

$$f_{\min} = \min(f_i) \quad (27)$$

where $i = 1, m$, then

$$\sigma_i = \tfrac{1}{2}(1 - sign(f_i)\frac{f_i}{f_{\min}} + (1 - sign(f_i))\frac{f_i}{f_{\max}}),\ 0 \le \sigma_i \le 1 \quad (28)$$

i.e.

$$\sigma_i = \begin{cases} \frac{f_i}{2}\left(\frac{1}{f_{min}} + \frac{1}{f_{max}}\right), \ f_i \geq 0 \\ 0, \ f_i < 0 \end{cases} \quad (29)$$

- For the method using the index NPWI

We calculate it by the formula:

$$NPWI = alb + tbp \quad (30)$$

where the values of albumin and total blood protein are normalized in advance, according to the formula:

$$\overline{p_{k,j}} = \frac{p_{k,j}}{p_{k,j} + p_{k',j}} \quad (31)$$

$$\overline{p_{k',j}} = \frac{p_{k',j}}{p_{k,j} + p_{k',j}} \quad (32)$$

where $p_{k,j}$ is the absolute value of the respective features, measured in a given day (for example, in the day of start of the weaning), and $p_{k',j}$ the identical value, measured in the previous day (for example, the day before the weaning).

The purpose of normalization is to minimize the variance between the classes and to increase the distance between them.

If $NPWI \geq 1$, then the patient has readiness to weaning (he/she belongs to class ω_2), in the other case the patient belongs to class ω_1 (he/she does not have readiness to weaning).

For classification with the introduced by the author "nonpulmonary weaning index" its value $G(x_i)$ is compared to 1.

Let

$$g_i = G(x_i)\text{-}1 \quad (33)$$

$$g_{\max} = \max(g_i) \quad (34)$$

$$g_{\min} = \min(g_i) \quad (35)$$

where $i = \overline{1,m}$, then

$$\sigma_i = \tfrac{1}{2}(1 - (1 - sign(g_i))\frac{g_i}{g_{\min}} + sign(g_i)\frac{g_i}{g_{\max}}), \ 0 \leq \sigma_i \leq 1 \quad (36)$$

i.e.

$$\sigma_i = \begin{cases} \frac{g_i}{2}\left(\frac{1}{g_{min}} + \frac{1}{g_{max}}\right), \ g_i \geq 0 \\ 0, \ g_i < 0 \end{cases} \quad (37)$$

- For the method using the IF Voronoi Diagrams.

Let the set P consisting of n points $p_i \in P$ be given in the plane.

Let us denote by $p_i q'$ the orthogonal projection of the segment qp_i on the straight line defined by the points p_i and p_j, i.e.

$\prod_{p_i p_j} qp_i = p_i q'$.

Let us denote by $dist_{p_j}$ the distance from point p_j to the nearest edge in IFVD (Fig. 1).

Then by definition: Intuitionistic fuzzy Voronoi diagrams over an arbitrary point-set P in the plane is defined as the part of the plane, located in n cells. Each cell is unique for the point $p_i \in P$ and is such that point q lies in the cell, corresponding to the point p_i, if the following conditions are simultaneously fulfilled:

$$dist(q, p_i) < dist(q, p_j) \quad (38)$$

and

$$\prod_{p_i p_j} qp_i < dist_{p_i} \text{ when } q' \in p_i p_j \quad (39)$$

for every p_j from P and $j \neq i$.

If the condition (39) is not fulfilled, i.e.

$$\prod_{p_i p_j} qp_i > dist_{p_i} \text{ when } q' \in p_i p_j \quad (40)$$

then point q will lie inside the strip between the cells, corresponding to the point p_i and point p_j, i.e. point q will lie in the area of indeterminacy.

The concept of intuitionistic fuzzy modification of Voronoi diagrams, comes to transforming part of the plane to convex regions (Fig. 1) that are separated by strips. These strips define the areas of indeterminacy. The presence of such areas is especially suitable, even necessary, when dealing with classification problems in the field of medicine, since there are numerous cases in the medical practice when the patient cannot be diagnosed, or a particular treatment cannot be prescribed without additional examinations, consultations with specialists or waiting. In the terms of decision-making theory this corresponds to a situation under which no decision is made, i.e. the patient has not been classified. When using classification with *IFVD*, this means the pattern (vector, point) has fallen in the areas (strips) of indeterminacy.

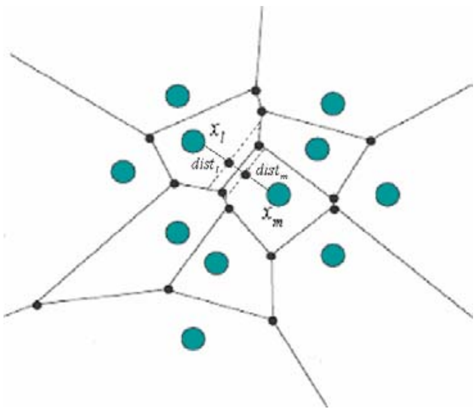

Fig. 1 Intuitionistic fuzzy modification of Voronoi diagrams

In this case the classification is done as follows:

1. Point z with unknown classification is associated to class ω_i, to which the point x_l with known classification belongs and in whose corresponding cell z falls.
2. If z falls in an area of indeterminacy, it is not classified

Let $i-$th region have k_i' neighbors from class 1 and k_i'' neighbors from class 2, where the classes are as the defined above. Then:

$$\sigma_i = \frac{k_i''}{k_i' + k_i''},\ 0 \le \sigma_i \le 1 \qquad (41)$$

An example

The obtained results from the application of the algorithm described above are summarized in Table 1.

Table 1

σ – SDA	σ – SLR	σ - NPWI	σ - IFVD	μ	ν	π
0.37332	0.41096	0.44789	0.5123	0.39022	0.52082	0.08896
0.34003	0.38534	0.38737	0.42333	0.35235	0.58809	0.05956
0.31066	0.34172	0.38176	0.43381	0.31414	0.6032	0.08266
0.29184	0.27983	0.3444	0.27487	0.25067	0.67476	0.07457
0.33422	0.40662	0.28331	0.36044	0.30729	0.62806	0.06465
0.37087	0.39355	0.18724	0.28203	0.2411	0.60655	0.15235
0.29641	0.3147	0.29088	0.45405	0.29602	0.6529	0.05108
0.35844	0.42365	0.30742	0.44309	0.29094	0.57024	0.13882
0.31298	0.36784	0.28525	0.33666	0.29024	0.61044	0.09932
0.26354	0.32124	0.5772	0.35334	0.30035	0.589	0.11065
0.25837	0.29683	0.61545	0.48048	0.2942	0.53389	0.17191
0.30473	0.33482	0.60035	0.50952	0.32307	0.48277	0.19416
0.29243	0.35198	0.68437	0.50143	0.34308	0.50581	0.15111
0.39693	0.43308	0.73336	0.5225	0.41373	0.4625	0.12368
0.29209	0.35479	0.79121	0.51404	0.33465	0.46708	0.19827
0.57289	0.65525	0.70386	0.63417	0.5998	0.31758	0.08262
0.47893	0.52707	0.72972	0.6182	0.49239	0.34709	0.16052
0.63078	0.72127	0.64446	0.71925	0.62805	0.29342	0.07853
0.70973	0.81464	0.70195	0.69857	0.68817	0.21276	0.09907
0.65149	0.71822	0.72999	0.69738	0.62245	0.24917	0.12838
0.72471	0.81739	0.64235	0.6833	0.686	0.23522	0.07878
0.6664	0.8003	0.69921	0.72191	0.66793	0.24582	0.08625
0.71054	0.82933	0.65196	0.63285	0.66133	0.23958	0.09909
0.65988	0.76567	0.59599	0.46883	0.58683	0.29714	0.11603
0.62853	0.71468	0.33484	0.50351	0.50581	0.35073	0.14346
0.70389	0.80727	0.22914	0.50973	0.47771	0.27962	0.24267
0.68593	0.79431	0.36997	0.52798	0.53445	0.25684	0.20871
0.62824	0.72428	0.2987	0.43532	0.45363	0.33973	0.20664
0.56298	0.6453	0.3619	0.39737	0.45476	0.41894	0.1263
0.52666	0.60633	0.39265	0.4581	0.43756	0.43318	0.12926

All 300 data are divided randomly in 30 groups of 10 data and a value corresponding to the mean of the values associated with each group is obtained as a result. Thus, the content of the columns of the Table 1 is the following:

- In the first column of the table are given the values of σ for SDA, calculated by formula (19);
- In the second column are given the values of σ for SLR, calculated by formula (28);
- In the third column are given the values of σ for NPWI, calculated by formula (36);
- In the fourth column are given the values of σ for IFVD, calculated by formula (41);
- In the fifth, sixth and seventh columns are given the values of the degree of membership, degree of non-membership and degree of indeterminacy for each of the patients, calculated by the proposed algorithm.

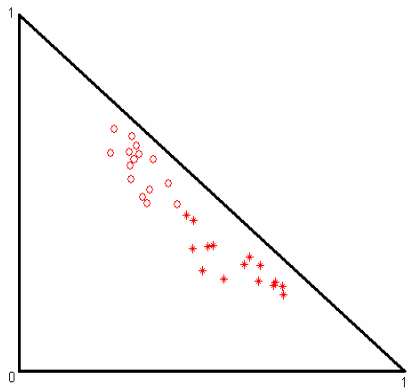

Fig. 2

Based on the values of μ and ν from Table 1 we construct the points in Fig 2. The objects from class 1 are denoted by circles and the objects from class 2 correspond to the crosses.

Conclusion

The application of the proposed algorithm allows the results of all the applied procedures used in the solution of a particular pattern recognition problem to be taken under consideration thus improving its objectivity. Because of that its use looks promising. In future work the authors will apply a second algorithm for the aggregation of the values received from the individual procedures. The results obtained from the two algorithms will be analyzed and compared.

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