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Noise Reduction of Measurement Data using Linear Digital Filters

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Summary: In this paper Butterworth, Chebyshev (Type I and II) and Elliptic digital filters are designed for signal noise reduction. On-line data measurements of substrate concentration from *E. coli* fed-batch cultivation process are used. Application of the designed filters leads to a successful noise reduction of on-line glucose measurements. The digital filters presented here are simple, easy to implement and effective - the used filters allow for a smart compromise between signal information and noise corruption.

Keywords: Measurement Data, Noise, Digital Filter Design.

1. INTRODUCTION

On-line measurements are difficult. This is due to the almost inexhaustible sources of disturbances. The sources may consist of, for instance, electromagnetic interference, hostile measuring environment, defective installation, insufficient maintenance or erroneous use and handling of the measuring system. Therefore, before any analysis of the measurements can be carried out, data screening is crucial. Corrupt measurements must be found and dealt with, so that false conclusions based on the measurements are avoided. Proper validation of data quality is essential to achieve reliable results. Corrupt data can be found and replaced/removed by different methods depending on the situation and the nature of the disturbance or fault. Computers make it possible to treat large amounts of data digitally and thereby the quality of the data increases.

In order to achieve good analysis results, the validation of the collected measurements is important. The measurement data must be representative for the investigated process. A continuous critical data validation gives information on the data quality and the condition of the measurement system. There are many different methods for data validation ranging from logical algorithms testing the reasonableness of the measurement to more sophisticated statistics or model based methods for outlier detection. Almost every measurement series is affected by:

- noise;
- missing values;
- outliers;
- drifting measurements or trends.

Noise is a common problem in almost every measurement system. The measurement noise is caused by electromagnetic disturbances, the design of the measuring devices, the methods used and so on, and is hard to avoid. The noise can be further amplified by incorrect installation, poor maintenance and changes in the ambient environment in which the measurement system is located. Irregularities within the process can also be considered as noise. The process noise is the noise which cannot be explained by variations in the measurement or communication system. The process noise may be caused by inhomogeneous mixing, random variations of, for instance, air bubbles and other non-measurable causes.

For noise reduction of measurement data filters can be applied [2, 3, 6]. Depending on where in the measurement system the filters are applied, they can be either analogue or digital. Analogue filtering is commonly used in sensor devices for basic noise reduction, while digital filtering is used to achieve a wide variety of outputs in computers. Digital filters allow for a smart compromise between signal information and noise corruption [2, 3, 6].

Digital filtering is a large discipline and only the basics for noise reduction purposes will be discussed in this paper. The main purpose of this research is to design Butterworth, Chebyshev (Type I and II) and Elliptic digital filters for noise reduction of on-line substrate measurement of *E. coli* fed-batch cultivation process.

2. DIGITAL FILTER DESIGN

A data signal normally has a mixture of different frequency components in it. The frequency contents of the signal and their powers can be obtained through operations such as the Fast Fourier Transform (FFT) [2, 3, 6]. A low-pass filter passes relatively low frequency components in the signal but stops the high frequency components. The so-called cutoff frequency divides the passband and the stopband. In other words, the frequency components higher than the cutoff frequency will be stopped by a low-pass filter. This type of filter is especially useful since the random errors involved in the raw position data obtained through reconstruction are characterized by relatively high frequency contents.

2.1. Butterworth filter

Butterworth filters are one of the most commonly used digital filters [3]. They are fast and simple to use. Since they are frequency-based, the effect of filtering can be easily understood and predicted. Choosing a cutoff frequency is easier than estimating the error involved in the raw data in the spline methods.

The behavior of a filter can be summarized by the so-called frequency response function, H_c. The frequency response function of the Butterworth low-pass filter has the following form [3]:

$$\left|H_c\left(j\omega\right)\right|^2 = \frac{1}{1+\left(j\omega / j\omega_c\right)^{2n}}$$

where $j=\sqrt{-1}$, ω – the frequency [rad/s], ω_c – the cutoff frequency [rad/s], and n – the order of the filter.

The frequency response of the Butterworth filter is maximally flat (has no ripples) in the passband, and rolls off towards zero in the stopband. Butterworth filters have a monotonically changing magnitude function with ω. The Butterworth is the only filter that maintains this same shape for higher orders (but with a steeper decline in the stopband) whereas other varieties of filters (Chebyshev, Elliptic) have different shapes at higher orders [2, 6].

Compared with a Chebyshev Type I/Type II filter or an Elliptic filter, the Butterworth filter has a slower roll-off, and thus will require a higher order to implement a particular stopband specification. However, Butterworth filter will have a more linear phase response in the passband than the Chebyshev Type I/Type II and Elliptic filters.

2.2. Chebyshev Type I/Type II filter

Chebyshev filters are analog or digital filters having a steeper roll-off and more passband ripple than Butterworth filters. Chebyshev filters have the property that they minimize the error between the idealized filter characteristic and the actual over the range of the filter, but with ripples in the passband [2, 6].

Because of the passband ripple inherent in Chebyshev filters, filters which have a smoother response in the passband but a more irregular response in the stopband are preferred for some applications.

Type I Chebyshev Filters

These are the most common Chebyshev filters. The gain (or amplitude) response as a function of angular frequency ω of the n-th order low pass filter is [2, 6]:

$$G_n(\omega) = \left|H_n(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2\left(\frac{\omega}{\omega_0}\right)}},$$

where ε is the ripple factor, ω_0 is the cutoff frequency and $T_n()$ is a Chebyshev polynomial of the n-th order.

Type II Chebyshev Filters

Also known as inverse Chebyshev, this type is less common because it does not roll off as fast as type I, and requires more components. It has no ripple in the passband, but does have equiripple in the stopband. The gain is [2, 6]:

$$G_n(\omega,\omega_0) = \frac{1}{\sqrt{1+\frac{1}{\varepsilon^2 T_n^2(\omega/\omega_0)}}}$$

2.3. Elliptic filter

An elliptic filter (also known as a Cauer filter) is an electronic filter with equalized ripple (equiripple) behavior in both the passband and the stopband [2, 6]. The amount of ripple in each band is independently adjustable, and no other filter of equal order can have a faster transition in gain between the passband and the stopband, for the given values of ripple (whether the ripple is equalized or not). Alternatively, one may give up the ability to independently adjust the passband and stopband ripple, and instead design a filter which is maximally insensitive to component variations.

As the ripple in the stopband approaches zero, the filter becomes a type I Chebyshev filter. As the ripple in the passband approaches zero, the filter becomes a type II Chebyshev filter and finally, as both ripple values approach zero, the filter becomes a Butterworth filter.

The gain of a lowpass elliptic filter as a function of angular frequency ω is given by [2, 6]:

$$G_n(\omega) = \frac{1}{\sqrt{1+\varepsilon^2 R_n^2(\xi,\omega/\omega_0)}},$$

where R_n is the nth-order elliptic rational function (sometimes known as a Chebyshev rational function); ω_0 is the cutoff frequency; ε is the ripple factor; ξ is the selectivity factor.

The value of the ripple factor specifies the passband ripple, while the combination of the ripple factor and the selectivity factor specify the stopband ripple.

3. DIGITAL FILTERING OF ON-LINE SUBSTRATE MEASUREMENTS OF *ESCHERICHIA COLI* FED-BATCH CULTIVATION

The cultivation condition of the fed-batch cultivation of *Escherichia coli MC4110* and the experimental data have been published previously [1, 4] as a result of teamwork according to *DFG Project* with the *Institute of Technical Chemistry, University of Hannover*.

The on-line measurements of substrate (glucose) are presented in Fig. 1 [1, 4]. The average value (μ) and the standard deviation (σ^2) of the signal (glucose concentration) are: μ = 0.1410; σ^2 = 0.1444.

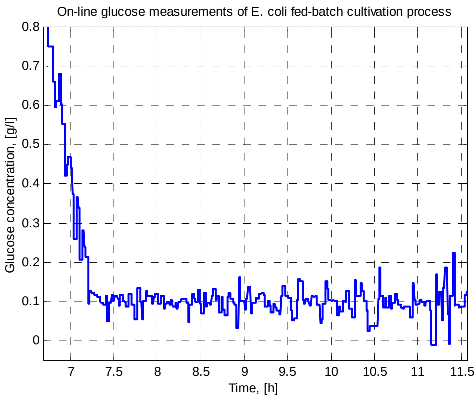

Fig. 1. Glucose measurements of *E. coli* fed-batch cultivation process

3.1. Butterworth digital filter design

[b, a] = **butter**(n, Wn)

designs an n-th order lowpass digital Butterworth filter and returns the filter coefficients in length n+1 vectors B (numerator) and A (denominator) [5].

The filter parameters are: n = 1; Wn = 0.005 and the filter returned [b, a] = [0.0078 0.0078 1.0000 -0.9844].

Noise reduction of on-line measurements of glucose for all used filters is fulfilled based on Matlab function "*filtfilt*" [5]. The function is zero-phase forward and reverses digital filtering.

Y = filtfilt(b, a, X) filters the data in vector X with the filter described by vectors A and B to create the filtered data Y. The filter is described by the difference equation:

y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
- a(2)*y(n-1) - ... - a(na+1)*y(n-na)

After filtering in the forward direction, the filtered sequence is then reversed and run back through the filter; Y is the time reverse of the output of the second filtering operation. The result has precisely zero phase distortion and magnitude modified by the square of the filter's magnitude response. Care is taken to minimize startup and ending transients by matching initial conditions [5].

3.2. Chebyshev type I filter design

[b, a] = **cheby1**(n, Rp, Wn)

designs an n-th order lowpass digital Chebyshev filter with Rp decibels of ripple in the passband. **cheby1** returns the filter coefficients in length n+1 vectors B (numerator) and A (denominator) [5].

For considered problem n = 1; Rp = 25; Wn = 0.1 and the filter result in [b, a] = [0.0088 0.0088 1.0000 -0.9823].

3.3. Chebyshev type II filter design

[b, a] = **cheby2**(n, Rs, Wn)

designs an n-th order lowpass digital Chebyshev filter with the stopband ripple Rs decibels down and stopband edge frequency Wn.

cheby2 returns the filter coefficients in length n+1 vectors B (numerator) and A (denominator) [5].

Here n = 1; Rs = 29; Wn = 0.2 and the filter returned
[b, a] = [0.0114 0.0114 1.0000 -0.9772].

3.4. Elliptic (Cauer) filter design

[b, a] = **ellip**(n, Rp, Rs, Wn)

designs an n-th order lowpass digital elliptic filter with Rp decibels of ripple in the passband and a stopband Rs decibels down. **ellip** returns the filter coefficients in length n+1 vectors B (numerator) and A (denominator) [5].

The Elliptic filter is designed at:

n = 1; Rp = 25; Rs = 20; Wn = 0.1 and result in
[b, a] = [0.0088 0.0088 1.0000 -0.9823].

The noise reduction of the glucose measurements based on the designed digital filters is presented in Fig. 2. For detailed presentation the same results are presented in Fig. 3. Here is more clearly are shown the effect of noise reduction of the considered on-line experimental data.

As it is shown on Fig. 2 there is a time delay when using these filters. This delay increases and becomes severe when significant noise reduction is desired. Digital filters of higher order, i.e. filters using more than one historic estimate to calculate the output, compensate for this to some extent, but there will always be a compromise between noise reduction and time lag. In off-line situations, where the filter does not have to be causal, it is possible to achieve high noise reduction with no time lag.

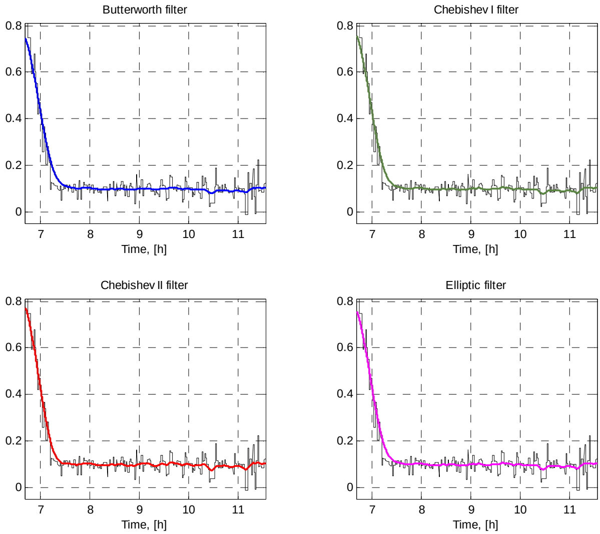

Fig. 2. Noise reduction of the glucose measurements based on Butterworth, Chebishev (I and II) and Elliptic digital filters

The resulting signals have the following characteristics:

- considering full signal (Fig. 2):

$\mu_{\text{Butterworth}} = 0.1427;$ $\sigma^2_{\text{Butterworth}} = 0.1325;$

$\mu_{\text{Chebishev I}} = 0.1423;$ $\sigma^2_{\text{Chebishev I}} = 0.1337;$

$\mu_{\text{Chebishev II}} = 0.1417;$ $\sigma^2_{\text{Chebishev II}} = 0.1357;$

$\mu_{\text{Elliptic}} = 0.1423;$ $\sigma^2_{\text{Elliptic}} = 0.1337.$

- considering part of signal (from 8h to 10 h from cultivation process) (Fig. 3):

$\mu_{(8\text{-}10)} = 0.0994;$ $\sigma^2_{(8\text{-}10)} = 0.0239;$

$\mu_{\text{Butterworth}(8\text{-}10)} = 0.0994;$ $\sigma^2_{\text{Butterworth}(8\text{-}10)} = 0.0024;$

$\mu_{\text{Chebishev I}(8\text{-}10)} = 0.0994;$ $\sigma^2_{\text{Chebishev I}(8\text{-}10)} = 0.0028;$

$\mu_{\text{Chebishev II}(8\text{-}10)} = 0.0994;$ $\sigma^2_{\text{Chebishev II}(8\text{-}10)} = 0.0036;$

$\mu_{\text{Elliptic}(8\text{-}10)} = 0.0994;$ $\sigma^2_{\text{Elliptic}(8\text{-}10)} = 0.0028.$

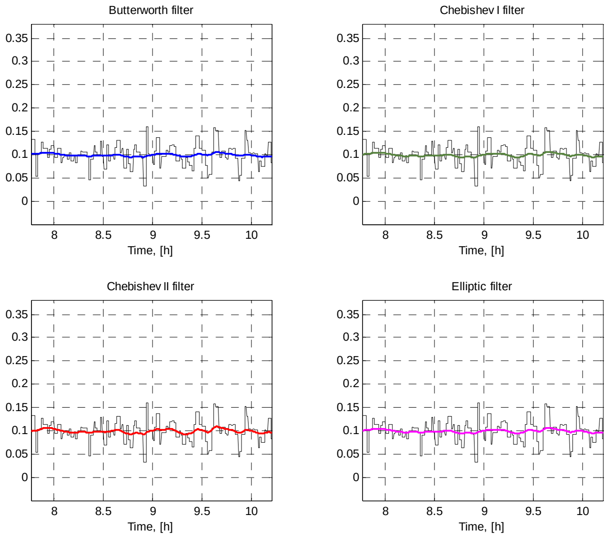

Fig. 3. Noise reduction of the glucose measurements based on Butterworth, Chebishev (I and II) and Elliptic digital filters (on a large scale)

Plots of the impulse response of the designed filters are presented in Fig. 4.

The results show that the designed filters reduced the measurement noise successfully. As it can be seen from starting values of μ and σ^2 (μ = 0.1410; σ^2 = 0.1444) after filtration of the signal are obtained a better result. For example application of digital Butterworth filter leads to $\mu_{\text{Butterworth(8-10)}}$ = 0.0994; $\sigma^2_{\text{Butterworth(8-10)}}$ = 0.0024.

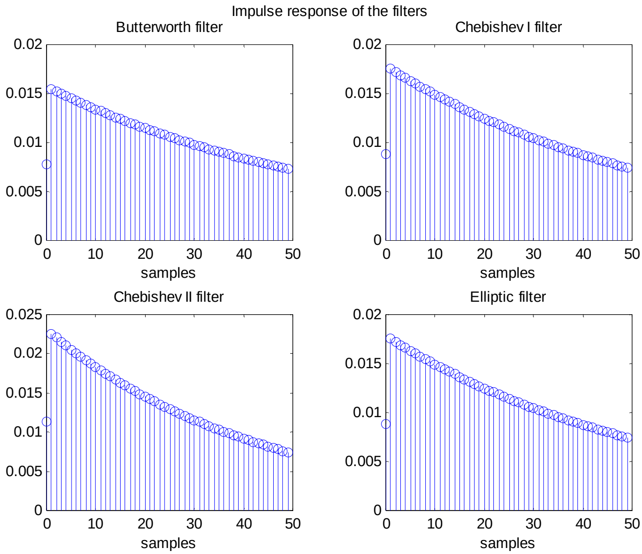

Fig. 4. Impulse response of the designed filters

5. CONCLUSION

For the noise reduction of on-line substrate measurements of *E. coli* fed-batch cultivation process digital filters are used. The measurement noise is normally of much higher frequency than the process variations themselves and, therefore, it is possible to filter the signal without losing too much information. The Buttrerworth, Chebishev (Type I and Type II) and Elliptic filters are designed. The digital filters presented here are simple but effective as they are easy to implement. The results show a successful noise reduction of on-line glucose measurements due to the fact that digital filters allow for a smart compromise between signal information and noise corruption.

Acknowledgements

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