



## **An Algorithm for Modified Times Series Analysis Method for Modeling and Prognosis of the River Water Quality**

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**Summary:** An algorithm and programs for modeling, analysis, and prognosis of river quality has been developed, which is a modified method of the time series analysis (TSA). The algorithm and program are used for modeling and prognosis of the river quality of Bulgarian river ecosystems.

**Keywords:** Times Series Analysis, Algorithm of Modified Times Series Analysis Method, River Water Quality.

### **1. INTRODUCTION**

The Commission to the Council and the European Parliament on implementation of Council Directive 91/676/EEC concerning the protection of waters against pollution gives that almost all Member States, with the exception of Italy, Portugal and Greece, provided at least some elements on assessment methods (simulation models and trend analysis) for the evaluation of trends in the water quality evolution.

Progress in the quality of action programs has been significant in the recent years and should contribute to the improvement of water quality in future reporting periods.

The TSA for water quality is a part of the retrospective modeling of water ecosystem functioning carried out with deterministic methods [1, 2]. The modified TSA is applied for modeling and prognosis of the Strouma river waters quality [6, 8].



In revolution investigations of the collective statistical treatment of the experimental data is developed and an analysis of the Strouma river waters quality in the Bulgarian part [7].

The aim of this work is developing and generalization of the algorithms and programs of the modified TSA.

## 2. A MODIFIED TIME SERIES ANALYSIS METHOD

For modeling of water pollution, a method of TSA is a determined component (trend) –  $\mathbf{x}_T$  describing the regularity of the development of the examined phenomenon, periodical component –  $\mathbf{x}_P$  and stochastic variable –  $\varepsilon_t$  [1, 2]:

$$\mathbf{x} = \mathbf{x}_T + \mathbf{x}_P + \varepsilon_t \quad (1)$$

where  $\mathbf{x}$  – the vector of the investigated indexes of the pollution.

The determined component (trend) –  $\mathbf{x}_T$  is a polynomial of 1<sup>st</sup> to 3<sup>rd</sup> degrees and the periodical component –  $\mathbf{x}_P$  is described by the order of Fourier.

In contrast to the conventional method of TSA the determined component as is a polynomial to high degrees:

$$\mathbf{x}_T = \sum_{j=0}^r a_j t^j \quad (2)$$

where  $a_j$  – coefficients of polynomial,  $j = 0, 1, 2, \dots, r$ ;  $r$  – degree of the polynomial.

The main trend shows the main tendencies in the alteration of the studied indices, and it is a straight line:

$$\mathbf{x}_T^M = A_0 + A_1 t \quad (3)$$

In contrast to the classical method for analyzing temporary series, where the Fourier series are used, the present research proposes the uses of the periodical functions from the type:



$$\mathbf{x}_p = \sum_{k=0}^p b_k \sin\left(\frac{2\pi}{c_k} t + d_k\right) \quad (4)$$

where:  $p$  – number of the periodical functions;  $b_k$ ,  $c_k$  and  $d_k$  – coefficients in the periodical functions,  $k = 0, \dots, p$ .

The number of the periodical functions  $p$  and the polynomial degree (3) is determined on basis the statistical criteria experimental Fisher function –  $F_E$  and experimental correlation coefficient –  $R^2_E$ .

Then the model (1) for analysis and prognosis has the following form ( $\varepsilon_t = 0$ ):

$$\mathbf{x} = \sum_{j=0}^r a_j t^j + \sum_{k=0}^p b_k \sin\left(\frac{2\pi}{c_k} t + d_k\right) \quad (5)$$

An algorithm and program is developed of the modified TSA.

### 2.1. The main algorithm of the Modified TSA

The generalized algorithm scheme is:

1. BEGINNING
2. Statistical treatment and analysis of main indexes for the water quality. Computing of statistical the experimental data. For this aim an algorithm and program. Output results from subroutine program are: the real and centered experimental data –  $x(i, j)$  and  $\tilde{x}(i, j)$ . The centered or the normalized data are computing is:

$$\tilde{x}_{i,j} = x_{i,j} - \bar{x}_j \text{ or } \tilde{x}_{i,j} = (x_{i,j} - \bar{x}_j) / \sqrt{s_j^2},$$

where:  $\bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{i,j}$ ;  $s_j^2 = \frac{1}{n-1} \sum_{i=1}^n (x_{i,j} - \bar{x}_j)^2$ ;  $n$  – number of

time partitions, ( $i = 1, \dots, n$ );  $m$  – number of investigation parameters of the pollutions, ( $j = 1, \dots, m$ ). Subprogram also makes the following: *check-ups for presence of trend, test for stationary, computing of autocorrelation, correlation matrix, and significance of correlation coefficients.*

3. The computing time in year [4, 5]:  $t = Year + \frac{1}{12}(Month - 0.5)$ .



4. The coefficients  $A_0, A_1$  are determined in the main trend (3), and the coefficients  $a_j, j = 0, \dots, r$  in trend (2). The procedure is carried out with the help of the subroutine POLRG from IMSL library of FORTRAN.
5. The experimental values of the periodical function:  $\mathbf{x}_p = \mathbf{x} - \mathbf{x}_T$  are calculated.
6. The coefficients  $b_k, c_k$  and  $d_k$  in periodical functions (4) and their numbers are determined. Comparison according to Fisher function –  $F$ . The procedure is carried out with the help of the subroutine for nonlinear regression RNSQ from IMSL library of FORTRAN.
7. The models (5) are determined by the programs for SIMPLEX method for optimization. The optimization program for direct search of the minimum of the criteria. The minimization criterion represents the sum of the squares of weighed residues ( $SSWR$ ) [9]:

$$SSWR = \sum_{i=1}^n \frac{(x_i^m - x_i)^2}{w_i^2}, \text{ where: } w_i = \max(x_i, x_i^m).$$

8. Computing validation of the models: the correlation coefficient –  $R^2_E$ , the experimental value of the Fisher's function –  $F_E$  and the relative error  $S_L$ . The  $S_L$  is calculated by [10, 11]:

$$S_L = \sqrt{\frac{1}{(n-\nu)} \sum_{i=1}^n \left( \frac{x_i^e - x_i^m}{x_i^e} \right)^2},$$

where  $x_i^e$  are the experimental values,  $x_i^m$  are the values calculated with the help of the model;  $\nu$  – degrees of freedom;  $n$  – number of the experimental data points.

9. Comparison with the theoretical correlation coefficient  $R^2_T$  and Fisher's function  $F_T$  is made. IF  $R^2_E > R^2_T$  and  $F_E < F_T$ , then the model is adequate. The lower the values of the relative error  $S_L$  are the higher the validity of the model for the investigated indexes.
10. Write:  $R^2_E, F_E, T'$  and  $S_L$ .
11. Write: Time,  $x_M^T$  from (2),  $x_T$  from (3), model for analysis and prognosis from (5).
12. END



The main algorithm is shown in Fig. 1, where: POLRG is a procedure of the polynomial regression, RNSQ is a procedure for nonlinear regression, SIMPLEX is a procedure for optimization. SIMPLEX serves for specifying of the model coefficients, if it is necessary.

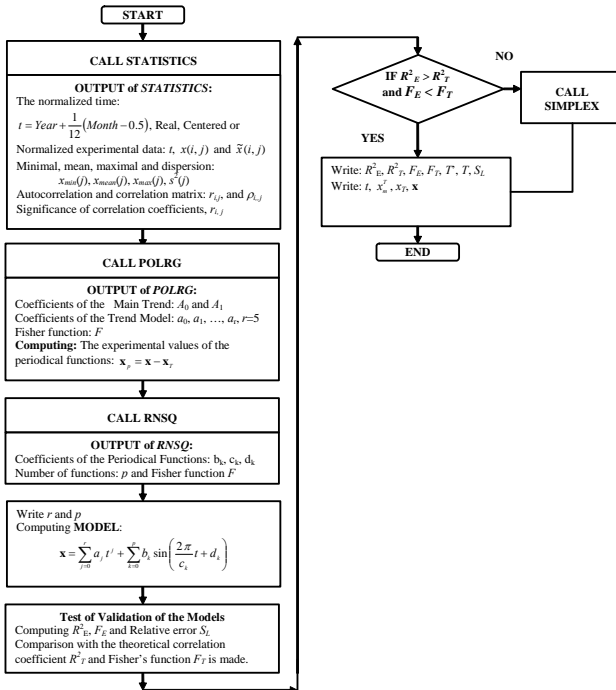


Fig. 1. Main algorithm of modified TSA

## 2.2. Algorithm for statistical treatment data

The block scheme of the algorithm is shown in Fig. 2, where: SPLINE is a procedure for extrapolation of the experimental data in the time; STATIONARY is a procedure for check-up of the *stationary* of the experimental observations; TREND is a procedure for test for presence of a *trend* (check-up for normally distribution of the observations); CORR is a procedure for calculating of the auto-



and correlation functions; STUDNT is a procedure for determining of the correlation coefficients.

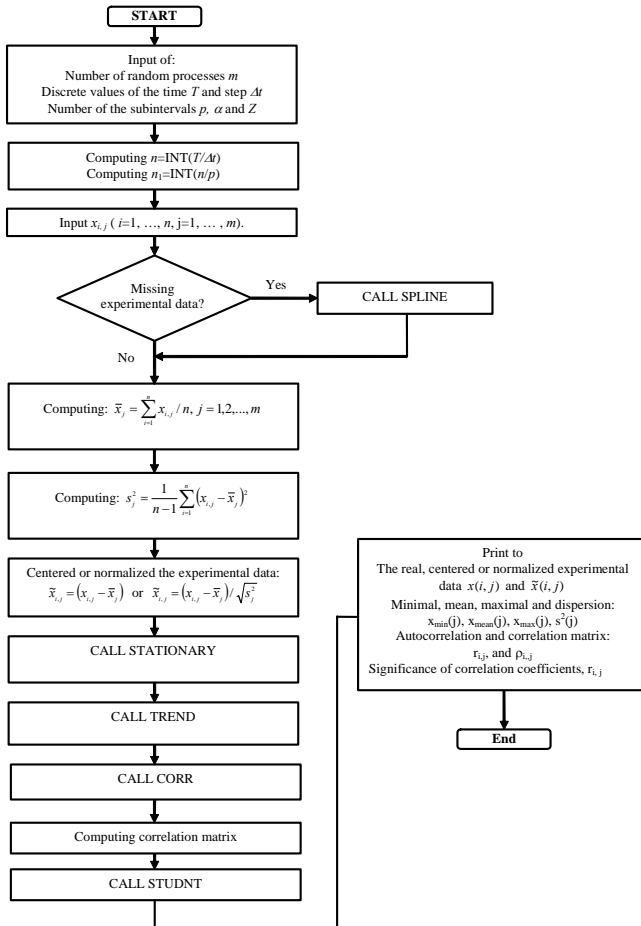


Fig. 2. Block scheme of the algorithm

All programs are developed on COMPAQ Visual FORTRAN Pro [3].



### 3. CONCLUSION

1. A model for analysis and prognosis of the river pollution, based on a modified time series analysis is developed. This approach can be used at modeling and prognosis of different water ecosystems. The proposed modified approach for time series analysis can be used on solving different problems connected with ecosystem management.
2. A method, algorithm and program for statistical treatment and analysis of experimental observation for the river water quality are developed. The statistical data treatment is used at modeling of the quality of the Struma river. The program can be used at the statistical data treatment also river ecosystems.
3. An algorithm and program for modified times series analysis is developed. Thus the offered algorithm and developed program are used for analysis, modeling and prognosis of the river water quality.

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