An Object Detection Method based on the Separability Measure using an Optimization Approach

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Abstract: *The detection of an object in an image can be set up as a maximization problem using separability measure as the objective function. The parameters of this objective function are the parameters used to define two regions in a mask. This mask has the same dimension as the image being considered. The two regions in the mask correspond to two regions in the image under investigation. The pixel value information in these two regions in the image would be used for the calculation of the separability measure. An optimization method would then be used to solve this maximization problem. This study demonstrated that the proposed method could detect the rectangle in the test image successfully. It showed that object detection and detailed segmentation could be carried out at the same time if the geometry of an object could be described mathematically.*

Keywords: *Object detection, Separability measure, Optimization method.*

Introduction

Threshold method is used to segment objects of interest in an image from the background. A common presentation of segmentation result is the use of a mask. A mask is a data file specifying for each pixel either one or zero. Value one indicates that the corresponding pixel is part of the object being segmented, whereas zero specifies a pixel in the background. A new measure of class separability has been demonstrated to be able to select the best mask (two regions representing respective object and background) from a set of masks [1]. Each of them is the different segmentation result of the same image. In this case, the measure is used for evaluation of segmentation results.

Point, line and edge detection and object recognition are key topics in image processing textbook [2]. Also, computer aided detection methods are frequently used in digital medical images. For example, breast cancer detection in mammograms [3]. The use of the new measure of class separability can be extended to detect an object in an image using an optimization approach. The objective function evaluates the new measure of separability for a given set of parameter values. Each set of parameter values refers to an enclosed region in an image that has a target object. The area outside the enclosed region of the image is called the outer region. The enclosed and outer regions represent the area of interest and background respectively. The calculation of the new measure of separability is based on pixels value in these two defined regions according to a given set of parameters. When the objective function is optimal, the location of the enclosed area defined by the corresponding set of parameters is expected to be near to the target object. Our objective of this study is to explore the application the newly developed measure of separability to the area of object detection using a simple test image.

Materials and method

Data

The test image has a small rectangle in white colour as shown in Figure 1. The dimension of this image is 50 pixels times 50 pixels and its background is black in colour. A pixel is a square and has an area of 1 unit square. Its top left corner and bottom right corner are at $(x_i - 1, y_i - 1, 0)$ and $(x_i, y_i, 0)$ respectively in a 3-dimensional co-ordinates system. The location of a pixel in the test image is simply denoted by $[x_i, y_i]$. The white rectangle is defined by 4 parameters, namely x_i, y_i, a and b. The centre of the white rectangle is at pixel $[x_i, y_i]$. The width of this rectangle is $(2a+1)$, i.e., a pixel both to the left and to the right from the centre. Similarly, the height of the rectangle is $(2b+1)$, i.e., b pixel both up and down from the centre. The rectangle in the test image has the respective values 22, 12, 2, 4 for x_i, y_i, a and b. The pixel values of the test image are stored in a matrix. For pixel $[x_i, y_i]$, its value is located at y_i th row and x_i th column of the matrix. Pixel values for black and white colours are 0 and 255 respectively.

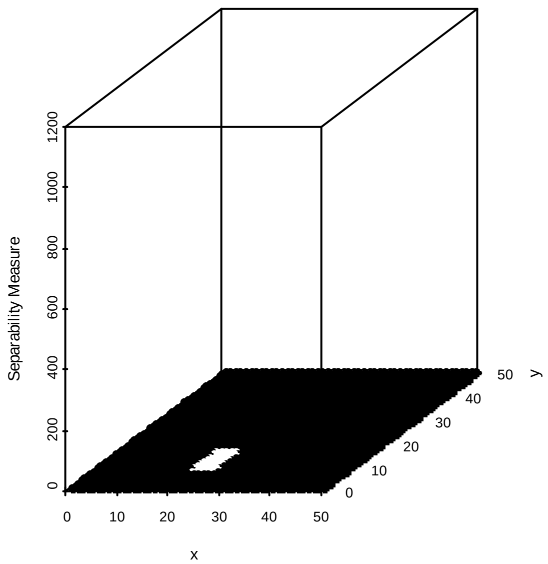

Fig. 1 The test image with a white rectangle and a background of black colour

Measure of separability

The new measure of separability is proposed by Choy and Hung [1]. It is based on the measure of separability used by Otsu as an image threshold method [4]. The new measure of separability is simply called separability measure and is briefly described. Let G_0 and G_1 denote the sets of gray levels corresponding to the pixel values in the background and in the object of an image respectively. In both G_0 and G_1, they have gray levels $0, 1, 2, \ldots, L$, where L is the maximum gray level and $L+1$ is the total number of possible gray levels. Let n_{0i} and n_{1i} be the number of pixels at level i for G_0 and G_1 respectively. Then the total number of pixels is:

$$N = \sum_{i=0}^{L} (n_{0i} + n_{1i}).$$

The respective probability distribution for the occurrence of gray level i for G_0 and G_1 are:

$$P_{0i} = \frac{n_{0i}}{N} \quad \text{and} \quad P_{1i} = \frac{n_{1i}}{N}, \quad i = 0, 1, 2, \ldots, L.$$

The zeroth- and first-order cumulative moments of the histogram for G_0 are defined respectively as:

$$\omega_{G_0} = \sum_{i=0}^{L} P_{0i} \quad \text{and} \quad \mu_{G_0} = \sum_{i=0}^{L} iP_{0i}.$$

Similarly,

$$\omega_{G_1} = \sum_{i=0}^{L} P_{1i} \quad \text{and} \quad \mu_{G_1} = \sum_{i=0}^{L} iP_{1i}$$

are the respective moments for G_1. The measure of class separability, σ_B^2, is defined as:

$$\sigma_B^2 = \frac{(\mu_T\ \omega_{G_0} - \mu_{G_0})^2}{\omega_{G_0}\ \omega_{G_1}}$$

where

$$\mu_T = \mu_{G_0} + \mu_{G_1} = \frac{1}{N}\sum_{i=0}^{L} i(n_{0i} + n_{1i})$$

is the mean gray level of the original matrix (or image).

The following four analyses were designed to detect and find the location of the white rectangle in the test image.

Analysis I

A mask is used to determine two regions in the test image. It has the same dimension as the test image. The rectangle in the mask is defined by four parameters described above. The parameters a and b was assigned to the value of 2 and 4 respectively. The values for x_i and y_i both range from 1 to 50. When part of a rectangle is outside the boundary of the mask, they would be truncated. For example, when x_i and y_i are both 1, the top left corner and bottom right corner of this rectangle are at pixels [1, 1] and [3, 4] respectively. That means area above and to the left of the centre of rectangle is truncated. Totally, 2500 different rectangles are formed. Each of them identifies two regions in the test image, i.e., pixels within the rectangle and pixels outside the rectangle. Pixels values within these two corresponding regions are used to calculate the separability measure. The maximum separability measure together with its rectangle centre location can be obtained from these 2500 solutions.

Analysis II

Instead of evaluating 2500 separability measures as shown in Analysis I, 100 random rectangle centres with pixel [x_i, y_i] are generated in this analysis. These 100 rectangles have a and b values equal to 2 and 4 respectively. Their separability measures are then calculated. The maximum separability measure together with its rectangle centre location can be obtained from these 100 solutions.

Analysis III

A mask that has the same dimension of the test image is used. It has a rectangle defined by four parameters (x_i, y_i, a and b) described above. So, it has two regions, i.e., area within the rectangle and area outside the rectangle. These two regions in the mask correspond to two regions in the test image. The separability measure of the test image can then be calculated based on the two regions identified by the mask. Hence, the detection of the rectangle in the test image can be set up as a maximization problem using separability measure as the objective function. The parameters of this objective function are the four parameters that define the rectangle in a mask. This objective function value varies as different masks are generated from different sets of rectangle parameters. An optimization method would then be used to solve this maximization problem. It is expected the objective function is optimal when the respective parameter values for the mask are 22, 12, 2 and 4. That means the rectangle in the mask corresponds exactly to the rectangle in the test image. The function called constrOptim from R was used to perform the optimisation [5]. By default constrOptim performs minimization. When the fnscale parameter was set to –1, it turns the problem into a maximization problem. The Nelder-Mead method was selected for the method parameter [6]. It uses only function values and is robust but relatively slow. The u_i and c_i parameters from the constrOptim were used to set constraints for the parameter x_i, y_i, a and b. These parameters were set to be greater than or equal to 0, 0, 1 and 1 respectively. Since the rectangle parameters were integers, the estimates returned from the constrOptim are rounded before they were used in the separability measure calculation. The 100 random set of parameters generated from Analysis II to define the rectangle in a mask are each used as the starting point for the constrOptim in this analysis.

Analysis IV

This analysis is similar to Analysis III except the mask is defined differently. The mask has two rectangles. The first one is defined by four parameters x_i, y_i, a and b described above. The centre of the second rectangle is the same as the first one. The area of second rectangle is twice the area of the first one. Therefore, the parameters for the second rectangle are x_i, y_i, $a+d$ and $b+d$, where

$$d = \frac{1}{4}\left(\sqrt{a^2 + 6ab + b^2} - (a+b)\right)$$

and the smallest integer not less than d should be taken to replace the original d. The area within first rectangle and the area between the two rectangles in the mask correspond to two regions in the test image for the calculation of the separability measure. The 100 random set of parameters generated from Analysis II to define the rectangle in a mask are each used as the starting point for the constrOptim in this analysis.

Results

The scatterplot3d function from R was used to show the separability measures, z_i of test image for a given mask or a mask determined by the optimization method. The vertical line was at the mid-point of the pixel that represents the centre of rectangle within a mask. The height of the lines corresponds to the numeric value of the measure. The co-ordinate at the top of the line is $(x_i - 0.5,\ y_i - 0.5,\ z_i)$.

In Analysis I, each separability measure value was shown in Fig. 2. This analysis showed the value of the measure of separability increased as the rectangle in the mask closed to the white rectangle in the test image. The measure of separability had a maximum when the mask was identical to the test image. That meant the mask correctly identified the white rectangle in the test image as one region and the background of the test image as another region. The separability measure had a maximum value of 1149.4 when the rectangle parameter values were 22, 12, 2 and 4. It was the optimal solution.

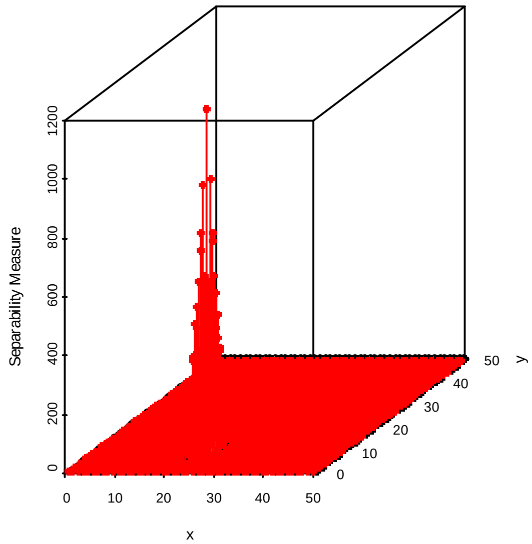

Fig. 2 The display of separability measure values calculated for each of the 2500 masks

In Analysis II, the separability measures associated to the 100 random masks were shown in Fig. 3. The separability measure had a maximum value of 316.5 when the rectangle parameter values were 23, 15, 2 and 4. This was not the optimal solution.

In Analysis III, the rectangle centre in these 100 masks were randomly generated However, the parameters *a* and *b* were both set to 2 assuming they were unknown. Each of these sets of rectangle parameters was used as the starting values for the constrOptim function to solve for the separability measure. Totally 70 solutions converged but only 4 of them were optimal. Their separability measures were shown in Fig. 4. The separability measure had the optimal value of 1149.4 (the longest vertical line) from four sets of starting points. Their respective rectangle centres were at pixels [44, 11], [30, 11], [27, 11] and, [23, 8]. The optimal solution has the parameter values 22, 12, 2 and 4. Twenty-five solutions indicated the 500 iterations limit for the Nelder-Mead method had been reached. The remaining five solutions indicated degeneracy of the Nelder-Mead simplex.

In Analysis IV, the rectangle centres of the 100 masks used here were same as the ones used in Analyses III. However, two rectangles were constructed in each mask. Each of these sets of rectangle parameters was used as the starting values for the constrOptim function to solve for the separability measure. Totally 99 solutions converged but only 1 of them were optimal. Their separability measures were shown in Fig. 5. When the starting point had rectangle centre at pixel [23, 15], the separability measure had the optimal value of 15390.5 (the longest vertical line in Fig. 5). This optimal value was different from the one obtained in Analysis III because the masks were defined differently. The optimal solution had the parameter values 22, 12, 2 and 4. The remaining one solution indicated the 500 iterations limit for the Nelder-Mead method had been reached.

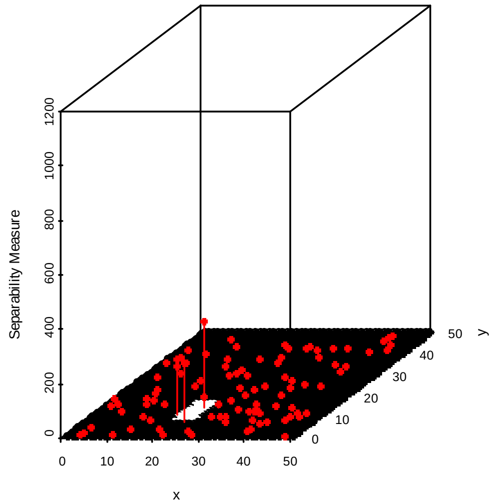

Fig. 3 The display of separability measure calculated for each of 100 random masks

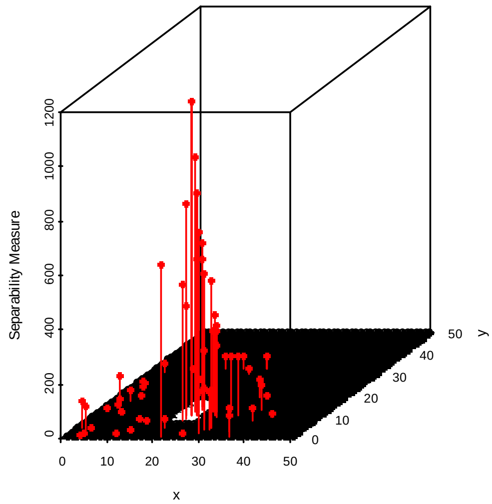

Fig. 4 The display of separability measures obtained from 70 solutions that converged

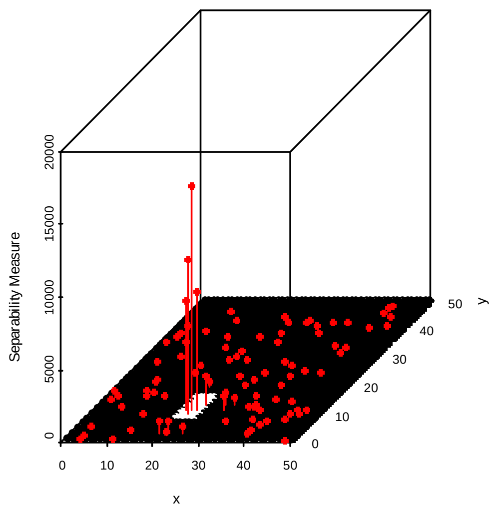

Fig. 5 The display of separability measures obtained from 99 solutions that converged

Discussion

The results from Analysis III and IV demonstrated that the proposed method could detect the rectangle in the test image successfully. Also, the mask used to identify two regions in the test image could have many definitions. The Nelder-Mead method was used to solve the maximization problem in this study. Other optimization technique such as particle swarm optimization could be considered [7]. This research potentially opens up a new methodology of object detection and segmentation using the separability measure in the framework of an optimization technique. The proposed method is generic. Other possible applications are to detect a missing boat in the sea or to detect an aircraft in the air. Object detection and detailed segmentation could be carried out at the same time if the geometry of an object could be described mathematically. Further development of this method is required when it is used to detect object embedded in a background of low contrast.

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