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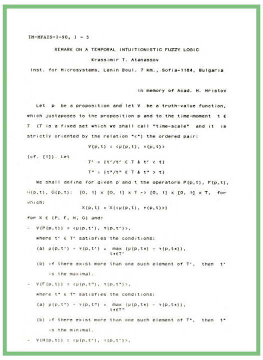
Remarks on a Temporal Intuitionistic Fuzzy Logic



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In the memory of Acad. H. Hristov

Let p be a proposition and let V be a truth-value function, which juxtaposes to the proposition p and to the time-moment $t \in T$ (T is a fixed set which we shall call "time-scale" and it is strictly oriented by the relation "<") the ordered pair (c.f. [1]).

$$V(p, t) = \langle \mu(p, t), \nu(p, t) \rangle$$

Let

$$T' = \{t' / t' \in T \text{ and } t' < t\}$$

$$T'' = \{t'' / t'' \in T \text{ and } t'' > t\}$$

We shall define for given p and t the operators

$$P, F, H, G\colon [0, 1] \times [0, 1] \times T \to [0, 1] \times [0, 1] \times T,$$

for which

$$X(p, t) = X(\langle \mu(p, t), \nu(p, t) \rangle)$$

for $X \in [P, F, H, G]$ and:

- $V(P(p, t)) = \langle \mu(p, t'), \nu(p, t') \rangle$, where $t' \in T'$ satisfies the conditions:

  (a) $\mu(p, t') - \nu(p, t') = \max\limits_{t^* \in T'} (\mu(p, t^*) - \nu(p, t^*))$,

  (b) if there exist more than one such element of T', then t' is the maximal.

- $V(F(p, t)) = \langle \mu(p, t''), \nu(p, t'') \rangle$, where $t'' \in T''$ satisfies the conditions:

  (a) $\mu(p, t'') - \nu(p, t'') = \max\limits_{t^* \in T''} (\mu(p, t^*) - \nu(p, t^*))$,

  (b) if there exist more than one such element of T'', then t'' is the minimal.

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– $V(H(p, t)) = \langle \mu(p, t'), \nu(p, t') \rangle$, where $t' \in T'$ satisfies the conditions:

(a) $\mu(p, t') - \nu(p, t') = \min_{t^* \in T'} (\mu(p, t^*) - \nu(p, t^*))$,

(b) if there exist more than one such element of T', then t' is the maximal.

– $V(G(p, t)) = \langle \mu(p, t''), \nu(p, t'') \rangle$, where $t'' \in T''$ satisfies the conditions:

(a) $\mu(p, t'') - \nu(p, t'') = \min_{t^* \in T''} (\mu(p, t^*) - \nu(p, t^*))$,

(b) if there exist more than one such element of T'', then t'' is the minimal (see e.g. [2, 3]).

Theorem 1: For every proposition p and for every time moment t:

(a) $V(H(p, t)) = V(N(P(N(p)), t)))$,

(b) $V(G(p, t)) = V(N(F(N(p)), t)))$.

Proof: (a) $V(N(F(N(p)), t))) = \langle \mu(p, t'), \nu(p, t') \rangle$

where t' is the maximal element of T' for which:

$$\nu(p, t') - \mu(p, t') = \max_{t^* \in T'} (\nu(p, t^*) - \mu(p, t^*)).$$

Therefore, t' is the maximal element of T' for which:

$$\mu(p, t') - \nu(p, t') = \min_{t^* \in T'} (\mu(p, t^*) - \nu(p, t^*)),$$

i.e.,

$$\langle \mu(p, t'), \nu(p, t') \rangle = V(H(p, t)).$$

(b) is proved analogically. □

Theorem 2: For every two propositions p and q, and for every time moment t:

(a) $H(p \supset q, t)) \supset (P(p, t) \supset P(q, t))$,

(b) $G(p \supset q, t)) \supset (F(p, t) \supset F(q, t))$,

(c) $N(P(N(p \supset q), t)) \supset (P(p, t) \supset P(q, t))$,

(d) $N(F(N(p \supset q), t)) \supset (F(p, t) \supset F(q, t))$.

are IFSs.

Proof: (a) $V(H(p \supset q, t) \supset (P(p, t) \supset P(q, t))) =$

$$= \mathrm{H}(\langle \max(\nu(p), \mu(q)), \min(\mu(p), \nu(q)), t \rangle) \supset \langle \mu(p, t_1), \nu(p, t_1) \rangle \supset \langle \mu(q, t_2), \nu(q, t_2) \rangle),$$

(where t_1 and t_2 are both maximal elements of T' for which the maximums of $\mu(p, t_1) - \nu(p, t_1)$ and of $\mu(p, t_2) - \nu(p, t_2)$ are achieved)

$$= \langle \max(\nu(p, t'), \mu(q, t')), \min(\mu(p, t'), \nu(q, t')) \rangle$$
$$\supset \langle \max \nu(p, t_1), \mu(q, t_2), \min(\mu(p, t_1), \nu(q, t_2)) \rangle,$$

(where t' is the maximal element of T' for which the maximum of $\max(\nu(p, t'), \mu(q, t')) - \min(\mu(p, t'), \nu(q, t'))$ is achieved)

$$= \langle \max(\nu(p, t_1), \mu(q, t_2)), \min(\mu(p, t'), \nu(q, t')), \min(\mu(p, t_1), \nu(q, t_2), \max(\nu(p, t'), \mu(q, t')) \rangle.$$

Then we consider the expression

$$a = \max(\nu(p, t_1), \mu(q, t_2)), \min(\mu(p, t'), \nu(q, t')) - \min(\mu(p, t_1), \nu(q, t_2), \max(\nu(p, t'), \mu(q, t')).$$

If there exist $t_2 \in T'$ for which $\mu(p, t_2) \geq \nu(q, t_2)$:

$$a \geq \mu(p, t_2) - \nu(q, t_2) \geq 0.$$

Let for every $t_2 \in T'$: $\mu(p, t_2) < \nu(q, t_2)$. Then, if there exist $t_1 \in T'$ for which $\nu(p, t_1) \geq \mu(q, t_1)$:

$$a \geq \nu(p, t_1) - \mu(q, t_1) \geq 0.$$

Let for every $t_1 \in T'$: $\nu(p, t_1) < \mu(q, t_1)$. Then, if there exist $t' \in T'$ for which $\min(\mu(p, t'), \nu(q, t')) \geq \max(\nu(p, t'), \mu(q, t'))$:

$$a \geq \min(\mu(p, t'), \nu(q, t')) - \max(\nu(p, t'), \mu(q, t')) \geq 0.$$

Let for every $t' \in T'$:

$$\min(\mu(p, t'), \nu(q, t')) < \max(\nu(p, t'), \mu(q, t'))$$

and let $t_0 \in T'$. If $\mu(p, t_0) \leq \nu(q, t_0)$, then $\nu(p, t_0) \leq \nu(q, t_0)$ and $\nu(q, t_0) < \mu(q, t_0)$, which is a contradiction.

If $\nu(p, t_0) < \mu(q, t_0)$, then $\mu(p, t_0) < \nu(q, t_0)$ and $\mu(q, t_0) < \nu(q, t_0)$, which is a contradiction. Therefore (a) is valid.

(b)-(d) are proved analogically. □

From the above definition, it follows the validity of:

Theorem 3: For every two propositions p and q, and for every time moment t:

(a) If $H(p, t)$ is an IFT, then $P(p, t)$ is an IFT;

(b) If $G(p, t)$ is an IFT, then $F(p, t)$ is an IFT;

From the equalities:

$$P(P(p, t), t) = P(p, t),$$

$$F(F(p, t), t) = F(p, t),$$

it follows the validity of:

Theorem 4: For every proposition p and for every time moment t:

(a) $P(P(p, t), t) \supset P(p, t)$

(b) $H(p, t) \supset H(H(p, t), t)$

are IFTs.

Let $W' = \{t' / t' \in T' \& t' \leq t\}$, $W'' = \{t'' / t'' \in T'' \& t'' \geq t\}$. The operators $\overline{P}$, $\overline{H}$, $\overline{F}$ and $\overline{G}$ are defined as the respective above, but for W' and W'' instead of T' and T''. For them the above assertions are valid also.

Theorem 5: For every two propositions p and q, and for every time moment t:

(a) If $\overline{H}(p, t)$ is an IFT, then $\overline{F}(p, t)$ and (p, t) are IFTs,

(b) If $\overline{G}(p, t)$ is an IFT, then $\overline{F}(p, t)$ and (p, t) are IFTs,

(c) If $\overline{H}(p, t) \& \overline{G}(p, t)$ is an IFT, then $\overline{P}(p, t) \vee (p, t) \vee \overline{F}(p, t)$ is an IFT,

(d) If $\overline{H}(p, t) \vee \overline{G}(p, t)$ is an IFT, then there exists $t^* \in T$ for which (p, t^*) is an IFT,

where (p, t) denotes the proposition p at the time-moment t.

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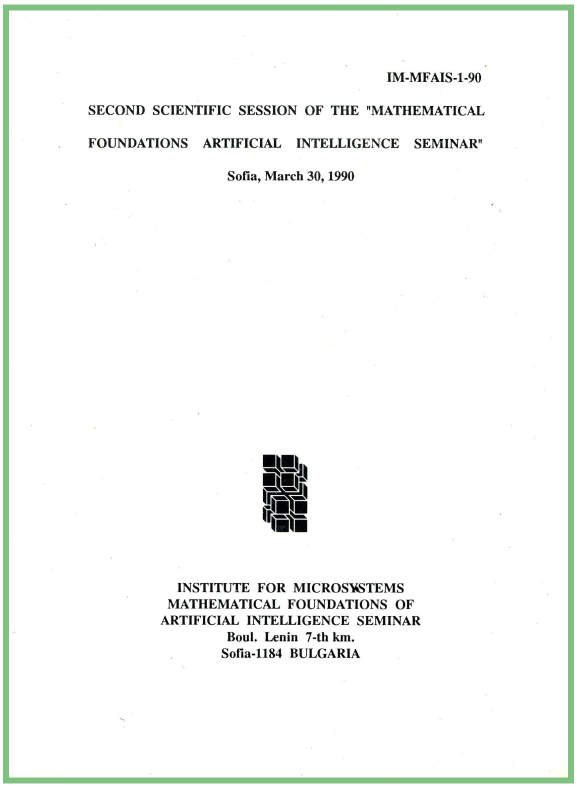
1. Atanassov K. Two Variants of Intuitionistic Fuzzy Propositional Calculus, Preprint IM-MFAIS-5-88, Sofia, 1988.

Facsimiles



IM-MFAIS-1-90

SECOND SCIENTIFIC SESSION OF THE "MATHEMATICAL FOUNDATIONS ARTIFICIAL INTELLIGENCE SEMINAR"

Sofia, March 30, 1990

INSTITUTE FOR MICROSYSTEMS
MATHEMATICAL FOUNDATIONS OF ARTIFICIAL INTELLIGENCE SEMINAR
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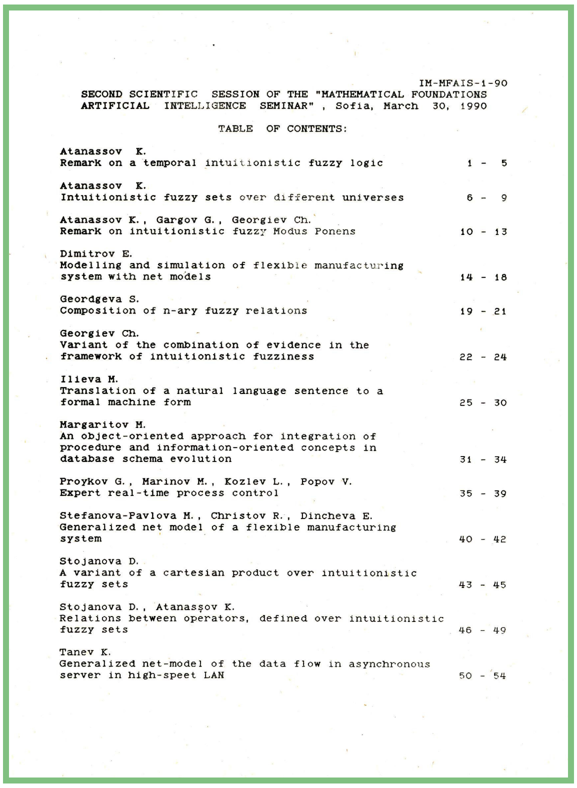

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SECOND SCIENTIFIC SESSION OF THE "MATHEMATICAL FOUNDATIONS ARTIFICIAL INTELLIGENCE SEMINAR", Sofia, March 30, 1990

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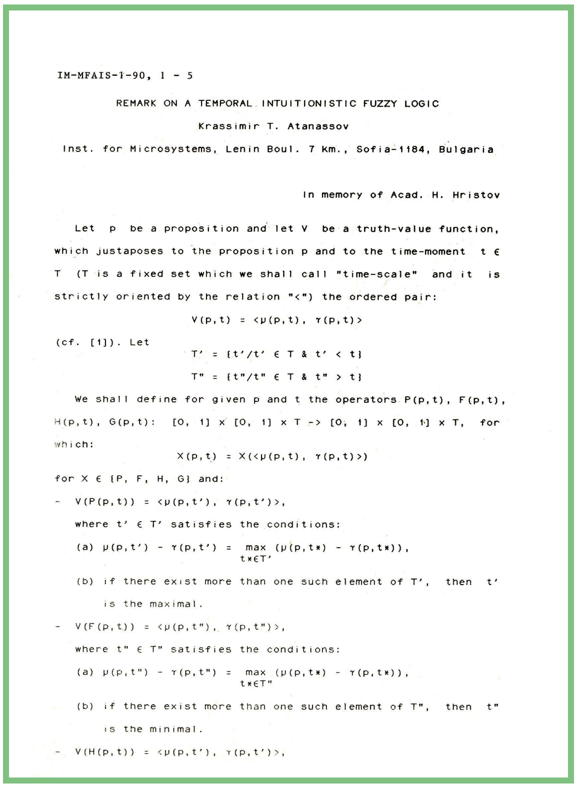

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REMARK ON A TEMPORAL INTUITIONISTIC FUZZY LOGIC

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In memory of Acad. H. Hristov

Let p be a proposition and let V be a truth-value function, which justaposes to the proposition p and to the time-moment $t \in T$ (T is a fixed set which we shall call "time-scale" and it is strictly oriented by the relation "<") the ordered pair:

$$V(p,t) = \langle \mu(p,t), \gamma(p,t) \rangle$$

(cf. [1]). Let

$$T' = \{t'/t' \in T \ \& \ t' < t\}$$

$$T'' = \{t''/t'' \in T \ \& \ t'' > t\}$$

We shall define for given p and t the operators P(p,t), F(p,t), H(p,t), G(p,t): $[0, 1] \times [0, 1] \times T \to [0, 1] \times [0, 1] \times T$, for which:

$$X(p,t) = X(\langle \mu(p,t), \gamma(p,t) \rangle)$$

for $X \in \{P, F, H, G\}$ and:

- $V(P(p,t)) = \langle \mu(p,t'), \gamma(p,t') \rangle$,

  where $t' \in T'$ satisfies the conditions:

  (a) $\mu(p,t') - \gamma(p,t') = \max_{t^* \in T'} (\mu(p,t^*) - \gamma(p,t^*))$,

  (b) if there exist more than one such element of T', then t' is the maximal.

- $V(F(p,t)) = \langle \mu(p,t''), \gamma(p,t'') \rangle$,

  where $t'' \in T''$ satisfies the conditions:

  (a) $\mu(p,t'') - \gamma(p,t'') = \max_{t^* \in T''} (\mu(p,t^*) - \gamma(p,t^*))$,

  (b) if there exist more than one such element of T", then t" is the minimal.

- $V(H(p,t)) = \langle \mu(p,t'), \gamma(p,t') \rangle$,

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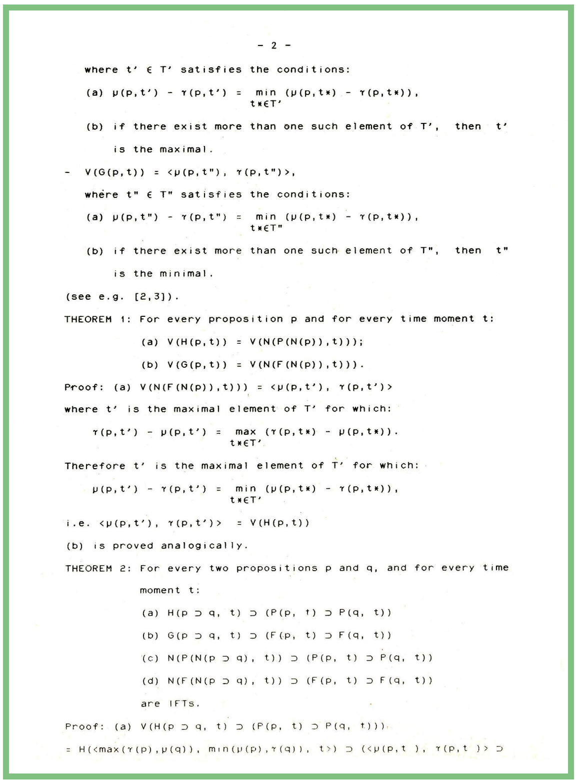

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where $t' \in T'$ satisfies the conditions:

(a) $\mu(p,t') - \gamma(p,t') = \min_{t^* \in T'} (\mu(p,t^*) - \gamma(p,t^*))$,

(b) if there exist more than one such element of T', then t' is the maximal.

- $V(G(p,t)) = \langle \mu(p,t''), \gamma(p,t'') \rangle$,

  where $t'' \in T''$ satisfies the conditions:

  (a) $\mu(p,t'') - \gamma(p,t'') = \min_{t^* \in T''} (\mu(p,t^*) - \gamma(p,t^*))$,

  (b) if there exist more than one such element of T", then t" is the minimal.

(see e.g. [2,3]).

THEOREM 1: For every proposition p and for every time moment t:

(a) $V(H(p,t)) = V(N(P(N(p)),t)))$;

(b) $V(G(p,t)) = V(N(F(N(p)),t)))$.

Proof: (a) $V(N(F(N(p)),t))) = \langle \mu(p,t'), \gamma(p,t') \rangle$

where t' is the maximal element of T' for which:

$$\gamma(p,t') - \mu(p,t') = \max_{t^* \in T'} (\gamma(p,t^*) - \mu(p,t^*)).$$

Therefore t' is the maximal element of T' for which:

$$\mu(p,t') - \gamma(p,t') = \min_{t^* \in T'} (\mu(p,t^*) - \gamma(p,t^*)),$$

i.e. $\langle \mu(p,t'), \gamma(p,t') \rangle = V(H(p,t))$

(b) is proved analogically.

THEOREM 2: For every two propositions p and q, and for every time moment t:

(a) $H(p \supset q, t) \supset (P(p, t) \supset P(q, t))$

(b) $G(p \supset q, t) \supset (F(p, t) \supset F(q, t))$

(c) $N(P(N(p \supset q), t)) \supset (P(p, t) \supset P(q, t))$

(d) $N(F(N(p \supset q), t)) \supset (F(p, t) \supset F(q, t))$

are IFTs.

Proof: (a) $V(H(p \supset q, t) \supset (P(p, t) \supset P(q, t)))$

$= H(\langle \max(\gamma(p),\mu(q)), \min(\mu(p),\gamma(q)), t \rangle) \supset (\langle \mu(p,t\ ), \gamma(p,t\ ) \rangle \supset$

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$\langle \mu(p,t_2),\ \nu(p,t_2) \rangle)$

(where t_1 and t_2 are both maximal elements of T' for which the maximums of $\mu(p,t_1) - \nu(p,t_1)$ and of $\mu(p,t_2) - \nu(p,t_2)$ are achieved)

$= \langle \max(\nu(p,t'),\mu(q,t')),\ \min(\mu(p,t'),\nu(q,t')) \rangle \supset \langle \max(\nu(p,t_1),$

$\mu(q,t_2)),\ \min(\mu(p,t_1),\ \nu(q,t_2)) \rangle$

(where t' is the maximal element of T' for which the maximum of $\max(\nu(p,t'),\mu(q,t')) - \min(\mu(p,t'),\nu(q,t'))$ is achieved)

$= \langle \max(\nu(p,t_1),\ \mu(q,t_2)),\ \min(\mu(p,t'),\nu(q,t')),$

$\min(\mu(p,t_1),\ \nu(q,t_2)),\ \max(\nu(p,t'),\ \mu(q,t')) \rangle.$

Then we consider the express

$a = \max(\nu(p,t_1),\ \mu(q,t_2)),\ \min(\mu(p,t'),\nu(q,t')) -$

$\min(\mu(p,t_1),\ \nu(q,t_2)),\ \max(\nu(p,t'),\ \mu(q,t')).$

If there exists $t_2 \in T'$ for which $\mu(q,t_2) \geq \nu(q,t_2)$:

$a \geq \mu(q,t_2) - \nu(q,t_2) \geq 0.$

Let for every $t_2 \in T'$: $\mu(q,t_2) < \nu(q,t_2)$. Then,

if there exists $t_1 \in T'$ for which $\nu(p,t_1) \geq \mu(p,t_1)$:

$a \geq \nu(p,t_1) - \mu(p,t_1) \geq 0.$

Let for every $t_1 \in T'$: $\nu(p,t_1) < \mu(p,t_1)$. Then,

if there exists $t' \in T'$ for which $\min(\mu(p,t'),\nu(q,t')) \geq$ $\max(\nu(p,t'),\ \mu(q,t'))$:

$a \geq \min(\mu(p,t'),\nu(q,t')) - \max(\nu(p,t'),\ \mu(q,t')) \geq 0.$

Let for every $t' \in T'$:

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$$\min(\mu(p,t'),\nu(q,t')) < \max(\nu(p,t'),\ \mu(q,t'))$$

and let $t_0 \in T'$. If $\mu(p,t_0) \leq \nu(q,t_0)$, then $\nu(p,t_0) < \nu(q,t_0)$ and $\nu(q,t_0) < \mu(q,t_0)$, which is a contradiction.

If $\nu(p,t_0) < \mu(p,t_0)$, then $\mu(q,t_0) < \nu(q,t_0)$ and $\mu(p,t_0) < \nu(p,t_0)$, which is a contradiction. Therefore (a) is valid.

(b) and (c) are proved analogically.

From the above definition follows the validity of

THEOREM 3: For every two propositions p and q, and for every time moment t:

(a) If H(p,t) is an IFT, then P(p,t) is an IFT;

(b) If G(p,t) is an IFT, then F(p,t) is an IFT;

From the equolities:

$$P(P(p,t),t) = P(p,t),$$

$$F(F(p,t),t) = F(p,t)$$

follows the validity of

THEOREM 4: For every proposition p and for every time moment t:

(a) $P(P(p,t),t) \supset P(p,t)$

(b) $H(p,t) \supset H(H(p,t),t)$

are IFTs.

Let

$$W' = \{t'/t' \in T \ \&\ t' \leq t\}$$

$$W'' = \{t''/t'' \in T \ \&\ t'' \geq t\}$$

The operators $\bar{P}$, $\bar{H}$, $\bar{F}$ and $\bar{G}$ are defined as the respective above, but for W' and W" instead of T' and T". For them the above assertions are valid also.

THEOREM 5: For every two propositions p and q, and for every time moment t:

(a) If $\bar{H}(p,t)$ is an IFT, then $\bar{F}(p,t)$ and (p,t) are IFTs;

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(b) $p \supset (p \veebar q)$,

(c) $(p \veebar q) \supset (q \veebar p)$,

(d) $(p \supset q) \supset ((r \veebar p) \supset (r \veebar q))$.

Proof: (d) $V((p \supset q) \supset ((r \veebar p) \supset (r \veebar q)))$

$= (\langle a,b \rangle \to \langle c,d \rangle) \to (\langle \max(a,e),\ \min(b,f) \rangle \to \langle \max(c,e),\ \min(d,f) \rangle)$

$= \langle \max(b,d),\ \min(a,d) \rangle \to \langle \max(c,e,\min(b,f)),\ \min(d,f,\max(a,e)) \rangle$

$= \langle \max(\min(a,d),c,e,\min(b,f)),\ \min(d,f,\max(a,e),\max(b,d)) \rangle$

and

$\max(\min(a,d),c,e,\min(b,f)) - \min(d,f,\max(a,e),\max(b,d))$

$\geq \max(\min(a,d),c,e) - \min(d,\max(a,e))$

if $a \geq d$:

$= \max(d,c,e) - \min(d,\max(a,e)) \geq 0;$

if $a < d$:

$= \max(a,c,e) - \min(d,\max(a,e))$

$\geq \max(a,e) - \min(d,\max(a,e)) \geq 0.$

(a)- (c) are proved analogically.

THEOREM 2.2 (a) $\Box(p \supset q) \supset (\Box p \supset \Box q)$ is an IFT,

(b) $\Box p \supset p$ is an IFT.

Proof: (a) $V(\Box(p \supset q) \supset (\Box p \supset \Box q))$

$= \Box\langle \max(b,c),\ \min(a,d) \rangle \supset (\langle a,1-a \rangle \supset \langle c,1-c \rangle)$

$= \langle \max(b,c),\ 1-\max(b,c) \rangle \supset \langle \max(1-a,c),\ \min(a,1-c) \rangle$

$= \langle \max(1-\max(b,c),1-a,c),\ \min(a,1-c,\max(b,c)) \rangle$

and

$\max(1-\max(b,c),1-a,c) - \min(a,1-c.\max(b,c))$

$= 2.\max(1-\max(b,c),1-a,c) - 1$

Let us assume that:

$$\max(1-\max(b,c),1-a,c) < 1/2.$$

Hence: $\max(b,c) > 1/2$, $1-a < 1/2$ and $c < 1/2$. Therefore: $b > 1/2$ and $a > 1/2$ which is a contradiction. Hence

$$\max(1-\max(b,c),1-a,c) \geq 1/2,$$

with which (a) is proved.

(b) is proved directly.

THEOREM 2.3: The following assertions are IFTs:

(a) $\neg\Box p \equiv \Diamond\neg p$,

(b) $\Box p \equiv \neg\Diamond\neg p$,

(c) $\neg\Diamond p \equiv \Box\neg p$,

(d) $\Diamond p \equiv \neg\Box\neg p$.

(e) $p \supset \Diamond p$,

(f) $\Box p \supset \Diamond p$.

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