

# **Intuitionistic Fuzzy Sets over Different Universes**

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In [1, 2] three variants of the operation "Cartesian product" were defined, in the general case, the operands in these variants of operations can be IFSs over different universes. AM other operations (see [1]) are defined in such way, that their operands are defined only over one universe.

Below we shall define operations with operands, defined over different universes.

Let E and F be two different universes, and let  $A_E$  and  $B_E$  be IFSs over E and F, respectively, i.e.,

$$A_E = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E \},\$$

$$B_F = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in F\}.$$

The IFS A, which is defined over the universe E, we shall call an "E-IFS". The operations over A and B are defined by:

$$A_E = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in E \},$$

$$A_E \cap B_F = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle \mid x \in E \cup F\},$$

$$A_E \bigcup B_F = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle \mid x \in E \bigcup F\},$$

$$A_E + B_F = \{ \langle x, \mu_A(x) + \mu_B(x) - \nu_A(x) \nu_B(x), \nu_A(x) \nu_B(x) \rangle \mid x \in E \cup F \},$$

$$A_E \cdot B_F = \{ \langle x, \mu_A(x) \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \nu_B(x) \rangle \mid x \in E \bigcup F \},$$

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where:

$$\begin{cases} \mu_A(x) = \begin{cases} \mu_A(x), & \text{if } x \in E \\ 0, & \text{if } x \in F - E \end{cases} \\ v_A(x) = \begin{cases} v_A(x), & \text{if } x \in E \\ 1, & \text{if } x \in F - E \end{cases} \end{cases}$$

and

$$\begin{cases} \mu_B(x) = \begin{cases} \mu_B(x), & \text{if } x \in F \\ 0, & \text{if } x \in E - F \end{cases} \\ v_B(x) = \begin{cases} v_B(x), & \text{if } x \in E \end{cases} \\ 1, & \text{if } x \in E - F \end{cases}$$

It can be seen directly, that all assertions from [1, 2] are valid here also. Obviously, for every universe E, every IFS over E is an E-IFS. On another hand every E-IFS can be interpreted as an ordinary IFS over the universe E.

We shall show that the elements of every set  $\sum = \{A_i / i \in I\}$ , with which we shall work, where  $A_i$  is an  $E_i$ -IFS ( $i \in I$  and I is a some index set), can be interpreted as ordinary IFSs over special universe.

Really, if  $card(\Sigma) = 1$ , the assertion is obviously valid. Let  $card(\Sigma) > 1$ . Then we can construct the universe

$$E^* = \bigcup_{i \in I} E_i$$

and the functions of membership and non-membership

$$\begin{cases} \mu_{A_i}^*(x) = \begin{cases} \mu_{A_i}(x), & \text{if } x \in E_i \\ 0, & \text{otherwise} \end{cases} \\ v_{A_i}^*(x) = \begin{cases} v_{A_i}(x), & \text{if } x \in E_i \\ 1, & \text{otherwise} \end{cases} \end{cases}$$

and hence every  $E_i$ -IFS  $A_i$  ( $i \in I$ ) will be an IFS over E. All defined above operations can be transformed over E.

The sense of E-IFSs consists in the possibility to work with IFSs for which for every element x of the universe, for functions  $\mu$  and  $\nu$  everywhere at least one of the inequations  $\mu_A(x) > 0$  and  $\nu_A(x) < 1$  for some A - a subset of E, is valid.



\* \*

It can be seen similarly to [3], that every rough set A [4, 5] for which the interval [0, 1] is a range of A and A can be represented by some IFS B. In this case  $\mu_B = \underline{A}$  and  $\nu_B = 1 - \overline{A}$ .

Here we shall show on Fig. 1 (from [5]) the relation between both above objects. We can use the sets  $X_1, ..., X_7$  as universes and we can construct the universe  $X = \bigcup_{i=1}^7 X_i$  as above with  $\mu_{B_i}$  and  $\nu_{B_i}$ :

$$\mu_{B_1}(x) = \begin{cases} 0.3, & \text{for } x \in X_1 \\ 0, & \text{for } x \in X - X_1 \end{cases}, \quad \nu_{B_1}(x) = \begin{cases} 0.6, & \text{for } x \in X_1 \\ 1, & \text{for } x \in X - X_1 \end{cases},$$

$$\mu_{B_2}(x) = \begin{cases} 0.4, & \text{for } x \in X_2 \\ 0, & \text{for } x \in X - X_2 \end{cases}, \quad \nu_{B_2}(x) = \begin{cases} 0.3, & \text{for } x \in X_2 \\ 1, & \text{for } x \in X - X_2 \end{cases},$$

etc., where  $B_i$  are  $E_i$ -IFS  $(1 \le i \le 7)$  and the IFS  $B = \bigcup_{i=1}^{7} B_i$ .

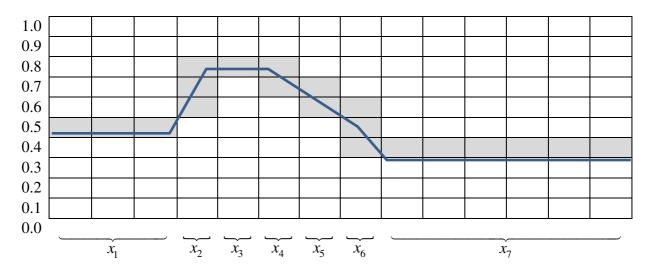


Fig. 1

On the other hand, every IFS B over an universe E, which has a representation  $B = \bigcup_{i=1}^{n} B_i$ , where  $B_i$  are  $E_i$ -IFS and with  $\mu_{B_i}(x) = \operatorname{const}_1 s$ ,  $\nu_{B_i}(x) = \operatorname{const}_2$  for every  $x \in E_i$   $(1 \le i \le n)$  can be represented by a some rough set with functions  $\underline{A}$  and  $\overline{A}$  in [0, 1].



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## **Facsimiles**

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Sofia, March 30, 1990

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## IM-MFAIS-1-90, 6 - 9 INTUITIONISTIC FUZZY SETS OVER DIFFERENT UNIVERSES Krassimir T. Atanassov Inst. for Microsystems, Lenin Boul. 7 km., Sofia-1184, Bulgaria In [1,2] three variants of the operation "Cartesian product" were defined. In the general case, the operands in these variants of operations can be IFSs over different universes. All other operations (see [1]) are defined in such way, that their operands are defined only over one universe. Below we shall define operations with operands, defined over Let E and F be two different universes, and let A and B be $\stackrel{E}{\text{E}}$ F IFSs over E and F, respectively, i.e. $A = \{ \langle \times, \ \mu_{A}(\times), \ \tau_{A}(\times) \rangle / \times \in E \},$ The IFS A, which is defined over the universe E we shall call "an E-IFS". The operations over A and B are defined by: $A = \{\langle x, \tau(x), \mu(x) \rangle / x \in E\};$ A ∩ S = (<x, min(μ (x), μ (x)), max(τ (x), τ (x))>/x∈E U F); $A \cup B = \{ \langle x, \max(y_{-}(x), y_{-}(x)), \min(Y_{-}(x), Y_{-}(x)) \rangle / x \in E \cup F \};$ A . B = ++x, $\mu$ (x). $\mu$ (x), $\tau$ (x)+ $\tau$ (x)- $\tau$ (x)- $\tau$ (x)-/x $\in$ E U F}, E F A B A B

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and hence every E -IFS A (i  $\in$  1) will be an IFS over E. All defined by the second contract of the second contract ifined above operations can be transformed over E. The sense of E-IFSs consists in the possibility to work with IFSs for which for every element  $\boldsymbol{x}$  of the universe, for functions  $\mu$  and  $\tau$  everywhere at least one of the inequations  $\mu$  (x)  $\rightarrow$  0 and and  $\tau$  (x) < 1 for some A - a subset of E, is valid. It can be seen similarly to [3], that every rough set A [4, 5] for which the interval [0, 1] is a range of A and A can be represented by some IFS B. In this case  $\mu$  = A and  $\tau$  = 1- A. Here we shall show on Fig. 1. (from [5]) the relation between both above objects. We can use the sets X ,  $\dots$  , X —as universes  ${\bf 1}$ and we can construct the universe  $X= \begin{bmatrix} 1 & 7 \\ 7 & X \\ 0 & X \end{bmatrix}$  as above with  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$  $\begin{array}{c} \mu \\ B \\ 2 \\ \end{array} \left( \begin{array}{c} (x) \ = \\ 0 \\ 0 \\ \end{array} \right. , \ \, \text{for } x \in X \\ = X \\ 2 \\ \end{array} \left( \begin{array}{c} x \\ (x) \ = \\ 0 \\ \end{array} \right. \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ 2 \\ 1 \\ \end{array} \right. , \ \, \text{for } x \in X - X \\ = X \\ \end{array} \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ 2 \\ 1 \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ 2 \\ 1 \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ 2 \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ 2 \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x \in X \\ \end{array} \right) \left( \begin{array}{c} 0.3, \ \, \text{for } x$ etc., where B are E - IFS (1 1 1 7) and the IFS B = U B. On the other hand, every IFS B over an universe E, which has a representation B = U B , where B are E -IFS and with  $\mu$  -(x) = Page 3 Page 4