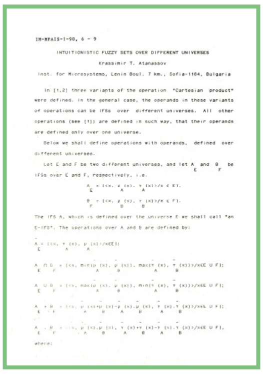

Intuitionistic Fuzzy Sets over Different Universes

Krassimir T. Atanassov*

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In [1, 2] three variants of the operation "Cartesian product" were defined, in the general case, the operands in these variants of operations can be IFSs over different universes. AM other operations (see [1]) are defined in such way, that their operands are defined only over one universe.

Below we shall define operations with operands, defined over different universes.

Let E and F be two different universes, and let A_E and B_E be IFSs over E and F, respectively, i.e.,

$$A_E = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

$$B_F = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in F\}.$$

The IFS A, which is defined over the universe E, we shall call an "E-IFS". The operations over A and B are defined by:

$$A_E = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in E\},$$

$$A_E \cap B_F = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle \mid x \in E \cup F\},$$

$$A_E \cup B_F = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid x \in E \cup F\},$$

$$A_E + B_F = \{\langle x, \mu_A(x) + \mu_B(x) - \nu_A(x)\nu_B(x), \nu_A(x)\nu_B(x)\rangle \mid x \in E \cup F\},$$

$$A_E \,.\, B_F = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle \mid x \in E \cup F\},$$

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where:

$$\begin{cases} \mu_A(x) = \begin{cases} \mu_A(x), & \text{if } x \in E \\ 0, & \text{if } x \in F - E \end{cases} \\ \nu_A(x) = \begin{cases} \nu_A(x), & \text{if } x \in E \\ 1, & \text{if } x \in F - E \end{cases} \end{cases}$$

and

$$\begin{cases} \mu_B(x) = \begin{cases} \mu_B(x), & \text{if } x \in F \\ 0, & \text{if } x \in E - F \end{cases} \\ \nu_B(x) = \begin{cases} \nu_B(x), & \text{if } x \in E \\ 1, & \text{if } x \in E - F \end{cases} \end{cases}.$$

It can be seen directly, that all assertions from [1, 2] are valid here also. Obviously, for every universe E, every IFS over E is an E-IFS. On another hand every E-IFS can be interpreted as an ordinary IFS over the universe E.

We shall show that the elements of every set $\sum = \{A_i / i \in I\}$, with which we shall work, where A_i is an E_i-IFS ($i \in I$ and I is a some index set), can be interpreted as ordinary IFSs over special universe.

Really, if card(Σ) = 1, the assertion is obviously valid. Let card(Σ) > 1. Then we can construct the universe

$$E^* = \bigcup_{i \in I} E_i$$

and the functions of membership and non-membership

$$\begin{cases} \mu^*_{A_i}(x) = \begin{cases} \mu_{A_i}(x), & \text{if } x \in E_i \\ 0, & \text{otherwise} \end{cases} \\ \nu^*_{A_i}(x) = \begin{cases} \nu_{A_i}(x), & \text{if } x \in E_i \\ 1, & \text{otherwise} \end{cases} \end{cases}$$

and hence every E_i-IFS A_i ($i \in I$) will be an IFS over E. All defined above operations can be transformed over E.

The sense of E-IFSs consists in the possibility to work with IFSs for which for every element x of the universe, for functions μ and ν everywhere at least one of the inequations $\mu_A(x) > 0$ and $\nu_A(x) < 1$ for some A – a subset of E, is valid.

*
*   *

It can be seen similarly to [3], that every rough set A [4, 5] for which the interval [0, 1] is a range of A and A can be represented by some IFS B. In this case $\mu_B = \underline{A}$ and $\nu_B = 1 - \overline{A}$.

Here we shall show on Fig. 1 (from [5]) the relation between both above objects. We can use the sets $X_1, \ldots, X_7$ as universes and we can construct the universe $X = \bigcup_{i=1}^{7} X_i$ as above with μ_{B_i} and ν_{B_i}:

$$\mu_{B_1}(x) = \begin{cases} 0.3, & \text{for } x \in X_1 \\ 0, & \text{for } x \in X - X_1 \end{cases}, \quad \nu_{B_1}(x) = \begin{cases} 0.6, & \text{for } x \in X_1 \\ 1, & \text{for } x \in X - X_1 \end{cases},$$

$$\mu_{B_2}(x) = \begin{cases} 0.4, & \text{for } x \in X_2 \\ 0, & \text{for } x \in X - X_2 \end{cases}, \quad \nu_{B_2}(x) = \begin{cases} 0.3, & \text{for } x \in X_2 \\ 1, & \text{for } x \in X - X_2 \end{cases},$$

etc., where B_i are E_i-IFS ($1 \le i \le 7$) and the IFS $B = \bigcup_{i=1}^{7} B_i$.

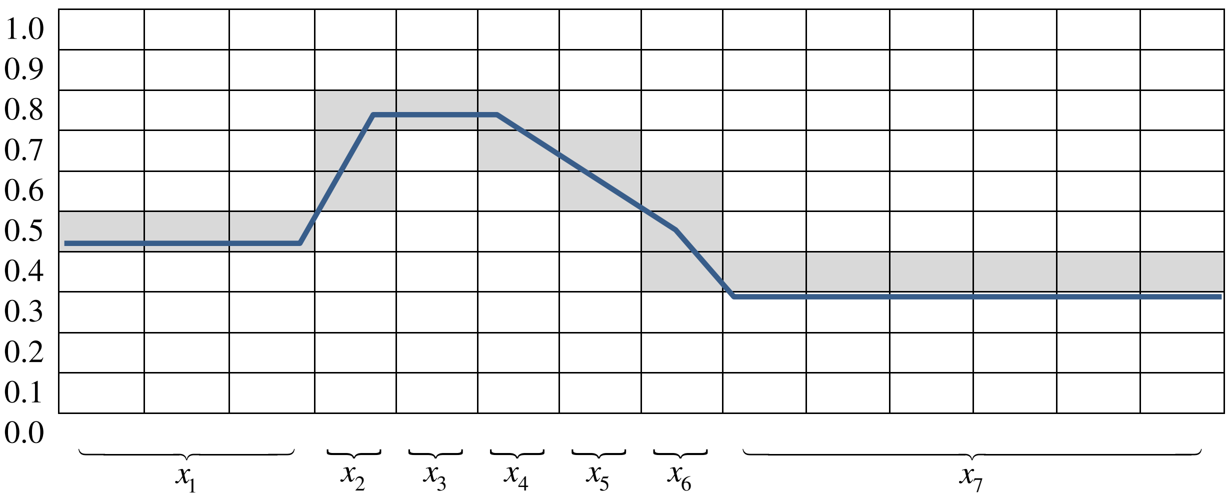

Fig. 1

On the other hand, every IFS B over an universe E, which has a representation $B = \bigcup_{i=1}^{n} B_i$, where B_i are E_i-IFS and with $\mu_{B_i}(x) = \text{const}_1 s$, $\nu_{B_i}(x) = \text{const}_2$ for every $x \in E_i$ ($1 \le i \le n$) can be represented by a some rough set with functions $\underline{A}$ and $\overline{A}$ in [0, 1].

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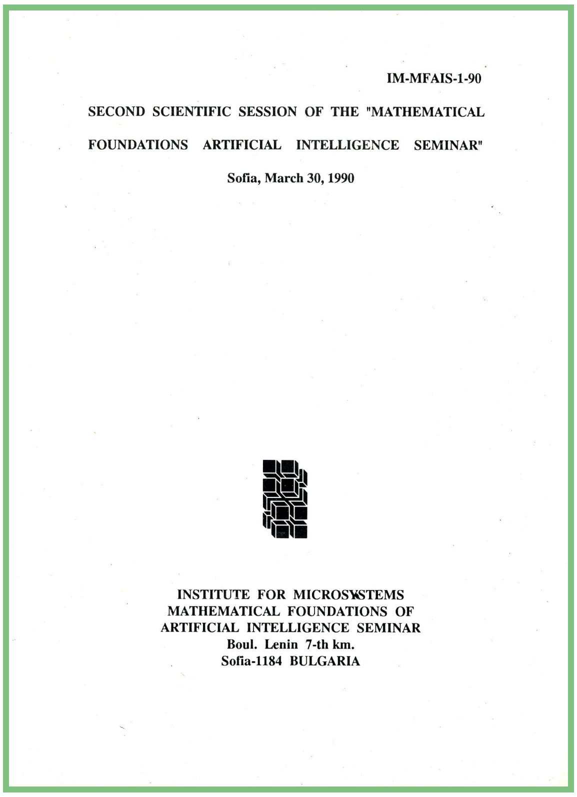
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Sofia, March 30, 1990

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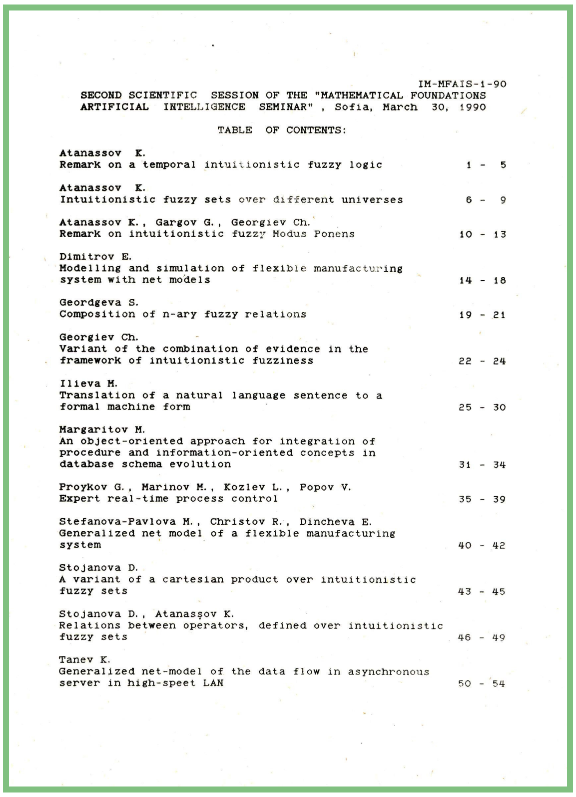
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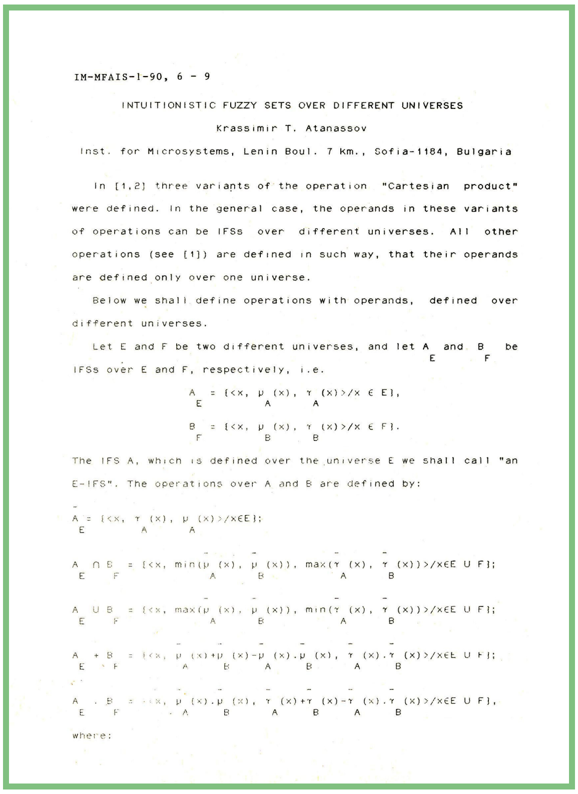
TABLE OF CONTENTS:

Contents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INTUITIONISTIC FUZZY SETS OVER DIFFERENT UNIVERSES

Krassimir T. Atanassov

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In [1,2] three variants of the operation "Cartesian product" were defined. In the general case, the operands in these variants of operations can be IFSs over different universes. All other operations (see [1]) are defined in such way, that their operands are defined only over one universe.

Below we shall define operations with operands, defined over different universes.

Let E and F be two different universes, and let A_E and B_F be IFSs over E and F, respectively, i.e.

$$A_E = \{<x, \mu_A(x), \gamma_A(x)>/x \in E\},$$

$$B_F = \{<x, \mu_B(x), \gamma_B(x)>/x \in F\}.$$

The IFS A, which is defined over the universe E we shall call "an E-IFS". The operations over A and B are defined by:

$$\bar{A}_E = \{<x, \gamma_A(x), \mu_A(x)>/x \in E\};$$

$$A_E \cap B_F = \{<x, \min(\bar{\mu}_A(x), \bar{\mu}_B(x)), \max(\bar{\gamma}_A(x), \bar{\gamma}_B(x))>/x \in E \cup F\};$$

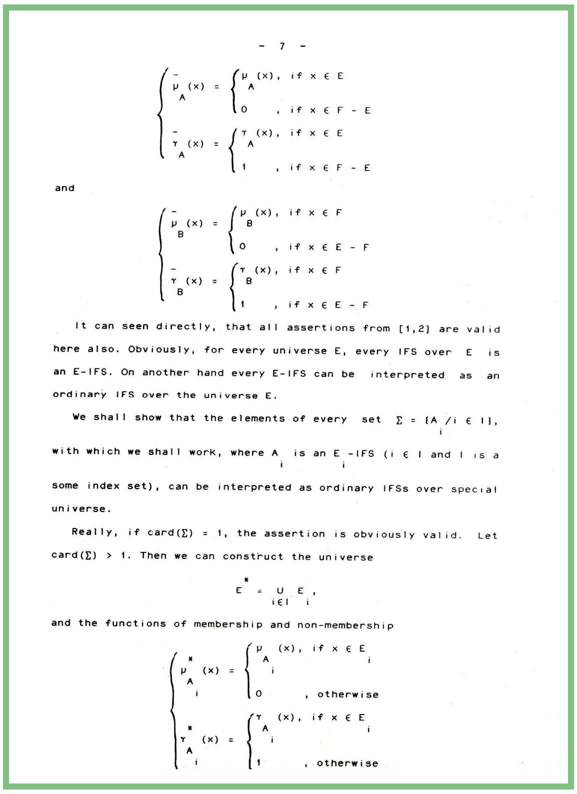
$$A_E \cup B_F = \{<x, \max(\bar{\mu}_A(x), \bar{\mu}_B(x)), \min(\bar{\gamma}_A(x), \bar{\gamma}_B(x))>/x \in E \cup F\};$$

$$A_E + B_F = \{<x, \bar{\mu}_A(x)+\bar{\mu}_B(x)-\bar{\mu}_A(x).\bar{\mu}_B(x), \bar{\gamma}_A(x).\bar{\gamma}_B(x)>/x \in E \cup F\};$$

$$A_E . B_F = \{<x, \bar{\mu}_A(x).\bar{\mu}_B(x), \bar{\gamma}_A(x)+\bar{\gamma}_B(x)-\bar{\gamma}_A(x).\bar{\gamma}_B(x)>/x \in E \cup F\},$$

where:

Page 1


- 7 -

$$\begin{cases} \bar{\mu}_A(x) = \begin{cases} \mu_A(x), & \text{if } x \in E \\ 0, & \text{if } x \in F - E \end{cases} \\ \bar{\gamma}_A(x) = \begin{cases} \gamma_A(x), & \text{if } x \in E \\ 1, & \text{if } x \in F - E \end{cases} \end{cases}$$

and

$$\begin{cases} \bar{\mu}_B(x) = \begin{cases} \mu_B(x), & \text{if } x \in F \\ 0, & \text{if } x \in E - F \end{cases} \\ \bar{\gamma}_B(x) = \begin{cases} \gamma_B(x), & \text{if } x \in F \\ 1, & \text{if } x \in E - F \end{cases} \end{cases}$$

It can seen directly, that all assertions from [1,2] are valid here also. Obviously, for every universe E, every IFS over E is an E-IFS. On another hand every E-IFS can be interpreted as an ordinary IFS over the universe E.

We shall show that the elements of every set $\Sigma = \{A_i / i \in I\}$, with which we shall work, where A_i is an E_i-IFS ($i \in I$ and I is a some index set), can be interpreted as ordinary IFSs over special universe.

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Page 2

INT. J. BIOAUTOMATION, 2016, 20(1), S69-S74

and hence every E_i-IFS A_i $(i \in I)$ will be an IFS over E. All defined above operations can be transformed over E.

The sense of E-IFSs consists in the possibility to work with IFSs for which for every element x of the universe, for functions μ and γ everywhere at least one of the inequations $\mu_A(x) > 0$ and $\gamma_A(x) < 1$ for some A - a subset of E, is valid.

* * *

It can be seen similarly to [3], that every rough set A [4, 5] for which the interval [0, 1] is a range of $\underline{A}$ and $\bar{A}$ can be represented by some IFS B. In this case $\mu_B = \underline{A}$ and $\gamma_B = 1 - \bar{A}$.

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$$\mu_{B_2}(x) = \begin{cases} 0.4, & \text{for } x \in X_2 \\ 0\ , & \text{for } x \in X - X_2 \end{cases} \qquad \gamma_{B_2}(x) = \begin{cases} 0.3, & \text{for } x \in X_2 \\ 1\ , & \text{for } x \in X - X_2 \end{cases}$$

etc., where B_i are E_i-IFS $(1 \le i \le 7)$ and the IFS $B = \bigcup_{i=1}^{7} B_i$.

On the other hand, every IFS B over an universe E, which has a representation $B = \bigcup_{i=1}^{n} B_i$, where B_i are E_i-IFS and with $\mu_{B_i}(x) =$

Page 3

$Const_1$, $\gamma_{B_i}(x) = Const_2$ for every $x \in E_i$ $(1 \le i \le n)$ can be represented by a some rough set with functions $\underline{A}$ and $\bar{A}$ in [0, 1].

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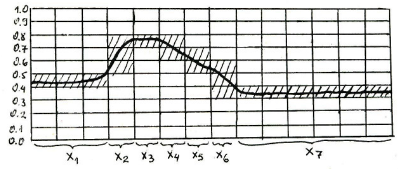

Fig. 1.

Page 4