Subtraction Procedure for Power-line Interference Removal from ECG Signals with High Sampling Rate

Georgy Mihov

Department of Electronics
Faculty of Electronic Engineering and Technologies
Technical University of Sofia
8 Kliment Ohridski Str., Sofia 1000, Bulgaria
E-mail: gsm@tu-sofia.bg

Received: February 15, 2018 **Accepted: May 11, 2018**

Published: June 30, 2018

Abstract: *The paper deals with some aspects of the subtraction procedure applied for power-line interference removing from high sampling rated ECG signals. Here high sampling rated ECG signal stands for signal with sampling rate-to-mains frequency ratio of about 15. Appropriate changes in the main stages of the subtraction procedure are introduced to ensure effective power-line interference removal. An adequate methodology is proposed to compensate the frequency deviation of the mains frequency. Besides, a specifically developed algorithm accelerates the initial procedure adaptation, as well as its work in case of abrupt changes of the mains frequency. Finally, program implementation of the modified subtraction procedure is elaborated and the results of simulated tests with 16 kHz sampling rated ECG signals are presented.*

Keywords: *ECG signals, Power-line interference removal, Subtraction procedure.*

Introduction

The subtraction procedure for power-line interference (PLI) removal from ECG signals has already proved its high efficiency (Levkov et al. [4], Christov and Dotsinsky [2], Dotsinsky and Daskalov [3]). Over the years, it was additionally investigated and improved demonstrating almost totally elimination of variable in PLI amplitude and frequency, what is more, without affecting the original ECG signals (Levkov et al. [5], Mihov and Dotsinsky [12], Mihov [9]).

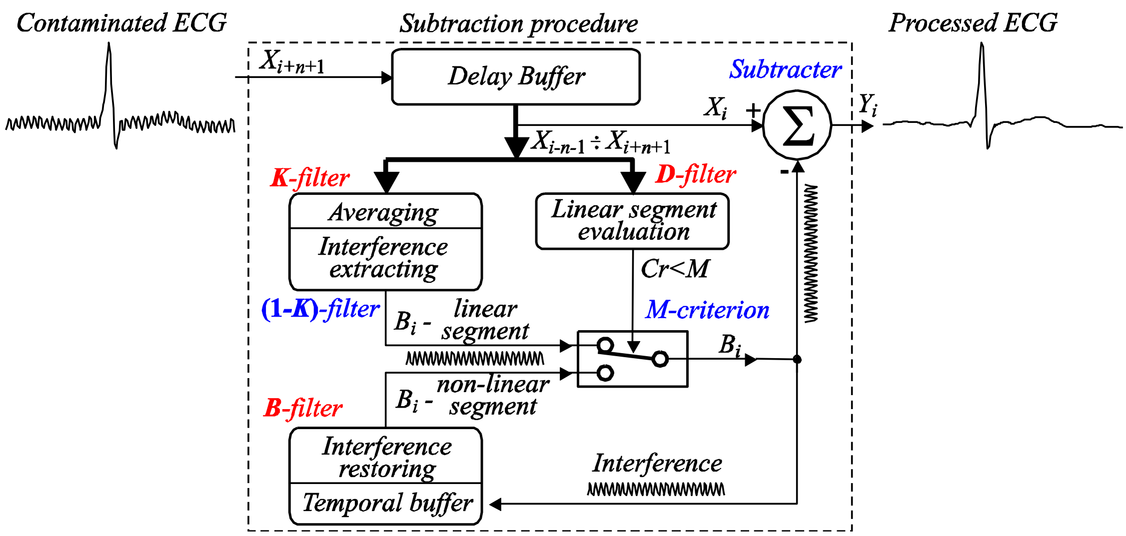


Fig. 1 Generalized structure of the subtraction procedure for power-line interference removal from ECG signals

The structure of the subtraction procedure (Fig. 1) consists of three main stages:

- *Linear segment evaluation*. Each ECG signal sample is tested whether it belongs to a linear segment (contaminated by interference). The linearity is evaluated by applying of appropriate criterion *Cr* (called **D-**filter), whose product must be less than a defined threshold *M* (*M-criterion*);
- *Interference extracting*. If the linearity criterion is fulfilled, the power-line interference within these segments is removed by means of a FIR filter (**K-**filter). The ongoing interference sample is obtained by subtracting its filtered value from the corresponding input sample; the procedure is denoted as (**1-K**)-filter. The calculated interference samples are currently stored in FIFO type temporal buffer.
- *Interference restoring*. When the linearity criterion is not fulfilled (non-linear segment is detected) an extrapolation procedure is performed (referred to as **B**-filter). It processes the stored FIFO values in order to evaluate the ongoing PLI value. The extrapolated interference value is subtracted from the ECG signal and is saved back into the buffer.

For diagnostic purposes, the ECG signals are usually sampled with a frequency of 250 to 500 Hz. For 50 Hz mains frequency (F), the ratio between sampling rate and F is between 5 and 10. Sometimes, for example when pacemaker's pulses have to be detected, a higher (over 5 kHz) sampling rate of the ECG signal is required. It may reach 16 to 128 kHz (Tsibulko et al. [15]); then the ratio between sampling rate and f becomes 320 through 2560. In such a case, it is necessary to introduce other appropriate approaches for performing the above mentioned stages of the subtraction procedure.

Special features in the main stages of the subtraction procedure caused by high ECG sampling rate

The ratio n between the constant sampling rate Q and the changeable PLI frequency F can be currently used as rounded integer number $n = \text{int}[Q/F]$. In the expected range of mains frequency deviation $\pm\,\Delta F_{max}$ around the rated value F_0, two rounded ratios within the mains period can be calculated: $n_{max} = \text{int}[Q/(F_0 - \Delta F_{max})]$ and $n_{min} = \text{int}[Q/(F_0 + \Delta F_{max})]$. The corresponding ratios related to the half-period of the deviated F are: $m_{max} = \text{int}[Q/(F_0 - \Delta F_{max})/2]$ and $m_{min} = \text{int}[Q/(F_0 + \Delta F_{max})/2]$.

Linear segment detection

An analysis of the linearity criterion (Mihov [6]) shows that the most accurate linear segment detection is done by complex criterion, formed as difference between the largest FD_{max} and the smallest FD_{min} first differences within the interval $[i - n, i + n]$ around the ongoing sample i (the array X contains the input ECG signal). To avoid the impact of the PLI, the first differences FD_i are taken from samples spaced by a period of interference, i.e. $FD_i = X_i - X_{i-n}$. Each sample is assumed to belong to linear segment if the requirement $|FD_{max} - FD_{min}| < M$ is met, where M is the threshold of the linearity criterion. A major drawback of the complex criterion is that the executing time is significant high and depends directly on the number of samples n within the PLI period.

This disadvantage is overcome by sequential implementation of the linearity criterion, introduced in Christov and Dotsinsky [2] and Dotsinsky and Daskalov [3]. Specifically applied to ECG signals with sampling rate $Q = 400$ Hz and PLI frequency $F = 50$ Hz, it uses the first two differences FD_i and FD_{i+2} checking n times the condition $|FD_{i+2} - FD_i| < M$.

When the number of samples within the PLI period is not integer, the simple first differences

FD_i must be replaced by the introduced in Mihov [8] complex first differences FD^*_i, which cover the full range of the F deviation

$$FD^*_i = \left(X_{i+m_{\min}} - X_{i-m_{\min}}\right)\left(1-k_d\right) + \left(X_{i+m_{\max}} - X_{i-m_{\max}}\right)k_d, \; k_d = \frac{\sin\dfrac{2m_{\min}\pi F}{Q}}{\sin\dfrac{2m_{\min}\pi F}{Q} - \sin\dfrac{2m_{\max}\pi F}{Q}}. \quad (1)$$

To estimate the linearity of the interval $[i-n, \; i+n]$ around the ongoing sample i, it is sufficient to confirm $2m_{\max} + 1 - m_{\min}$ times the linearity criterion proposed by Dotsinsky and Daskalov [3], e.g.

$$Cr \equiv \bigcap_{j=0}^{2m_{\max}-m_{\min}} \left(\left|FD^*_{i+m_{\max}+j} - FD^*_{i+j}\right| < M\right). \quad (2)$$

Here the time for checking the linearity criterion continues to be minimal since the inequality $|FD^*_{i+m_{\max}} - FD^*_i| < M$ is calculated once for the ongoing sample. The necessary condition for adequate complex first difference FD^*_i is $m_{\max} - m_{\min} \geq 1$. If $m_{\max} = m_{\min}$, it should be substituted $m_{\max} = m_{\min} + 1$.

Linear segment processing and interference extracting

The symmetrical averaging filters with first zero at the rated F_0 are the most convenient tools for linear segment processing and interference extracting. Mihov et al. [10] introduced an odd moving averaging **K**-filter with recurrent modification for all cases of even or odd multiplicity and non-multiplicity. The recurrent modification, the averaging filter and the transfer coefficient K_F are described for PLI frequency F by the following equations:

$$Y_i^* = \frac{Y_i - X_i K_F}{1-K_F}, \quad Y_i = \frac{1}{2m+1}\sum_{j=-m}^{m} X_{i+j}, \quad K_F = \frac{1}{2m+1}\frac{\sin\dfrac{(2m+1)\pi F}{Q}}{\sin\dfrac{\pi F}{Q}}. \quad (3)$$

Here $m = \text{int}[Q/(2F)]$ is the rounded to the less or equal integer number of samples within a half-period of the PLI, Y_i is the ongoing averaged sample, and Y^* belong to the array that contains the ECG signal processed by the modified filter. Y_i can be calculated using a pipe-lined procedure, which minimizes the computing time regardless of the number of averaged samples.

$$Y_i = Y_{i-1} + \frac{X_{i+m} - X_{i-m-1}}{2m+1}. \quad (4)$$

Interference restoring for non-linear segments

The interferences within non-linear segments are restored by means of the proposed by Mihov et al. [13] so-called **B**-filter, which is set up on the base of FIR filter with rectangular impulse response and is denoted as $\mathbf{K_B}$-filter. An use of such $\mathbf{K_B}$-filter build under the terms of Eq. (3) leads to the optimized equation for the extrapolated value B_i:

$$B_i = B_{i-(2m+1)} + (2m+1)K_{BF}\left(B_{i-m} - B_{i-m-1}\right), \quad K_{BF} = \frac{1}{2m+1}\frac{\sin\frac{\pi(2m+1)F}{Q}}{\sin\frac{\pi F}{Q}}. \quad (5)$$

In case of high ECG sampling rate, the extrapolation of the PLI is better to be done by a proposed by Mihov [9] reduced odd $\mathbf{K_B}$-filter.

$$K_{BF}B_{i-g(r-1)/2} = \frac{1}{k}\sum_{j=-r+1}^{0} B_{i+jg}, \quad K_{BF} = \frac{1}{r}\frac{\sin\frac{rg\pi F}{Q}}{\sin\frac{g\pi F}{Q}}. \quad (6)$$

This filter contains an odd number of samples *r* spaced by *g* samples of the basic filter. The reduction factor is $g = \text{int}[n/r]$. This application leads to optimized PLI extrapolated values:

$$B_i = B_{i-r.g} + r\,K_{BF}\left(B_{i-(r-1)g/2} - B_{i-(r+1)g/2}\right). \quad (7)$$

Compensation of a mains frequency deviation

The *F* deviation strongly influences the accuracy of the subtraction procedure, especially when non-linear ECG segments have to be processed. The proposed by Mihov et al. [14] PLI frequency compensation needs correction of the $\mathbf{K_B}$-, **K**- and **D**-filters consisting of recalculation of their transfer and modifying coefficients during the processing. These coefficients are interconnected, all of them being function of the sampling frequency *Q* and the interference frequency *F*. Therefore, the correction for every one filter is based on the currently recalculated new transfer coefficient K_{BFnew} of the $\mathbf{K_B}$-filter.

Currently correction the $\mathbf{K_B}$-filter

The transfer coefficient K_{BFnew} is calculated back from optimized equation for extrapolation.

Eq. (5) cannot be used in case of high ECG sampling rate because of the difference between two extremely closed power-line interference samples ($B_{i-m} - B_{i-m-1}$), which will introduce very often zero in the divider thus compromising the result of division. Therefore, the reduced $\mathbf{K_B}$-filter must be used and the coefficient K_{BFnew} has to be recalculated back by the optimized Eq. (7), e.g.

$$K_{BFnew} = \frac{B_i - B_{i-r.g}}{r\left(B_{i-(r-1)g/2} - B_{i-(r+1)g/2}\right)}. \quad (8)$$

*Repeated modification of **K**- and $\mathbf{K_B}$-filters*

If the used $\mathbf{K_B}$- and **K**-filters are dissimilar, both of them must be once more modified since their transfer coefficients K_{BF} and K_F are also different. Another possibility is to modify the $\mathbf{K_B}$-filter only, to define its new coefficient K_{BFnew}, after that to use it for K_{Fnew} calculation according to established functional dependence.

Within the expected deviation of the mains frequency $\pm\,\Delta F_{\max}$ around its rated value F_0, the transfer coefficient $K_B(F)$ of the $\mathbf{K_B}$-filter can be approximated by line (see Fig. 2) using the Descartes' equation $K_B(F) = K_B(F_0) + \tan\alpha(F - F_0)$, where $\tan\alpha = K_B^I(F)\big|_{F=F_0}$ and $K_B(F_0) \equiv K_B(F)\big|_{F=F_0}$.

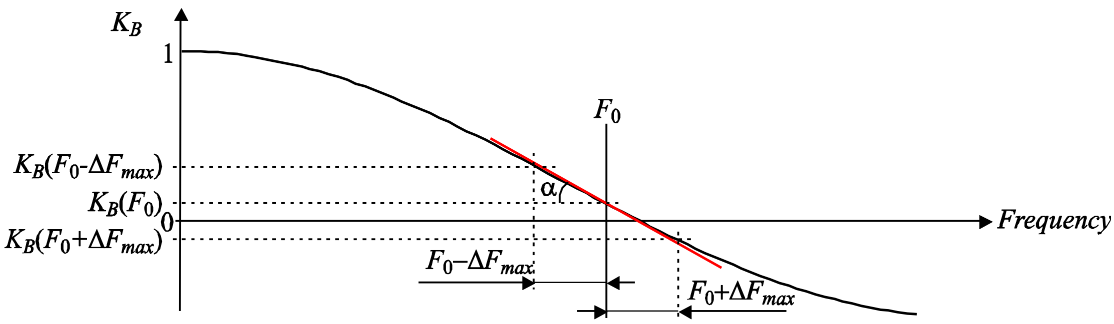


Fig. 2 Linear approximation of the frequency response of **K_B**-filter

The new coefficient K_{BFnew} of the **K_B**-filter at $F = F_{\mathrm{new}}$ can be expressed by

$$K_{BFnew} \approx K_B(F_0) + K_B^I(F)\big|_{f=F_0}(F_{new} - F_0), \quad K_{BFnew} \equiv K_B(F)\big|_{F=F_{new}}. \tag{9}$$

The transfer coefficient of the **K**-filter can also be presented as linear function inside the frequency deviation $F_0 \pm \Delta F_{\max}$. Analogously, the new coefficient K_{Fnew} is written as

$$K_{Fnew} \approx K(F_0) + K^I(F)\big|_{F=F_0}(F_{new} - F_0), \quad K_{Fnew} \equiv K(F)\big|_{F=F_{new}}. \tag{10}$$

A new expression about the coefficient K_{Fnew} can be obtained by defining the difference $(F_{new} - F_0)$ from Eq. (9) and replacing it in Eq. (10),

$$K_{Fnew} \approx K_{F0} + \frac{K^I(F)\big|_{F=F_0}}{K_B^I(F)\big|_{F=F_0}}\left[K_{BFnew} - K_{BF0}\right], \quad \left| \begin{array}{l} K_{F0} \equiv K(F_0) \equiv K(F)\big|_{F=F_0} \\ K_{BF0} \equiv K_B(F_0) \end{array} \right. . \tag{11}$$

Representing the first derivates of $K_B(F)$ and $K(F)$ as finite differences

$$K_B^I(F)\big|_{F=F_0} \approx \frac{K_B(F)\big|_{F=F_0+\Delta F_{\max}} - K_B(F)\big|_{F=F_0-\Delta F_{\max}}}{2\Delta F_{\max}} \text{ and } K^I(F)\big|_{F=F_0} \approx \frac{K(F)\big|_{F=F_0+\Delta F_{\max}} - K(F)\big|_{F=F_0-\Delta F_{\max}}}{2\Delta F_{\max}},$$

the new value of K_{Fnew} can be generalized as

$$K_{Fnew} \approx K_{F0} + R_{BK}(K_{BFnew} - K_{BF0}), \quad R_{BK} = \frac{K(F)\big|_{F=F_0+\Delta F_{\max}} - K(F)\big|_{F=F_0-\Delta F_{\max}}}{K_B(F)\big|_{F=F_0+\Delta F_{\max}} - K_B(F)\big|_{F=F_0-\Delta F_{\max}}}. \tag{12}$$

Correction of the sequential complex linearity criterion

The described in Eq. (2) current modification of the sequential linearity criterion is performed by recalculation of the coefficient k_d and the complex first differences **FD*** in Eq. (1). In the range of the expected mains frequency deviation $\pm\,\Delta F_{\max}$, the coefficient $k_d(F)$ can also be approximated by line. Analogously to the **K**-filter modification, the new coefficient

$k_{dnew} \equiv k_d\left(F\right)\Big|_{F=F_{new}}$ may be expressed by a similar to Eq. (12)

$$k_{dnew} \approx k_{dF0} + R_{dK}\left(K_{BFnew} - K_{BF0}\right), \quad R_{dK} = \frac{k_d\left(F\right)\Big|_{F=F_0+\Delta F_{\max}} - k_d\left(F\right)\Big|_{F=F_0-\Delta F_{\max}}}{K_B\left(f\right)\Big|_{F=F_0+\Delta F_{\max}} - K_B\left(F\right)\Big|_{F=F_0-\Delta F_{\max}}}, \tag{13}$$

where $k_{dF0} \equiv k_d\left(F\right)\Big|_{F=F_0}$. Eq. (13) represents the linear interpolation of the coefficient k_d within the expected frequency deviation from ($F_0 - \Delta F_{\max}$) to ($F_0 + \Delta F_{\max}$).

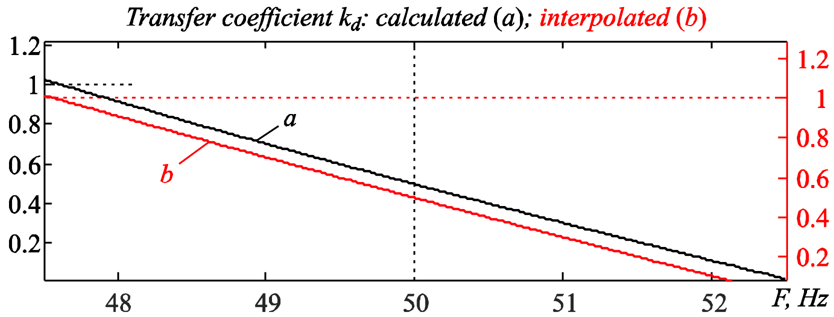


Fig. 3 Linear interpolation of the coefficient k_d for frequency deviation F = 50 ± 2.5 Hz

The result of the interpolation is shown in Fig. 3 for F = 50 ± 1.5 Hz and Q = 16 kHz using a reduced moving averaging $\mathbf{K_B}$-filter defined by Eq. (7) with parameters r = 3 and g = 106. The coincidence between the interpolated and the calculated transfer coefficient k_d is almost full (the two graphs are slightly shifted for better comparison).

Fig. 4 shows the result of applying the subtraction procedure on ECG signal with high sampling rate Q = 16000 Hz that is contaminated by rated mains frequency F_0 = 50 Hz (n = 320, m = 160) with deviation ΔF = ± 0.75 Hz (± 1.5%). The interference is extracted from the linear segments by averaging **K**-filter according to Eq. (3). This is performed by pipe-lined implementation of Eq. (4). The power-line interference for nonlinear ECG segments is restored through Eq. (7) with parameters r = 3 and g = 106. The sequential linearity criterion from Eq. (2) is applied with threshold M = 70 μV.

The initial value of the mains frequency is set to 50.75 Hz. In the middle of the tested epoch a steep transition of the mains frequency up to 49.25 Hz is simulated (see Fig. 4a). The first subplot presents the original signal, the second one shows the same signal superimposed by PLI and the third contains the processed ECG signal. The course and the switching of the linearity criterion as well as the absolute error committed can be seen in the fourth subplot. The last subplot shows the set frequency deviation (curve a – in green) and the result of the compensation (curve b – in black). The error in steady state does not exceed 30 μV.

A relatively difficult 'gripping' of the initially set mains frequency can be observed at the start of the subtraction procedure. Its reaction is faster during the next steep transition of the frequency.

Higher abrupt frequency changes hardly may be observed in practice as the power-supply is a relatively stable system. Still, such a jump was simulated in the contaminating signal of Fig. 4b. One may observe that the subtraction procedure needs more than 3.5 seconds to reach the steady state.

The velocity of following the frequency deviations may be optimized by introducing an adaptive threshold M. Such dynamic threshold R_t was proposed by Christov [1]. It is based on noise-to-signal ratio, which is defined as the sum S_{NL} of the nonlinear segments divided by the length S_E of the processed ECG epoch, e.g. $R_t = S_{NL} / S_E$. Firstly, a low initial R_t is selected; then its value rises remaining continuously near to 10%.

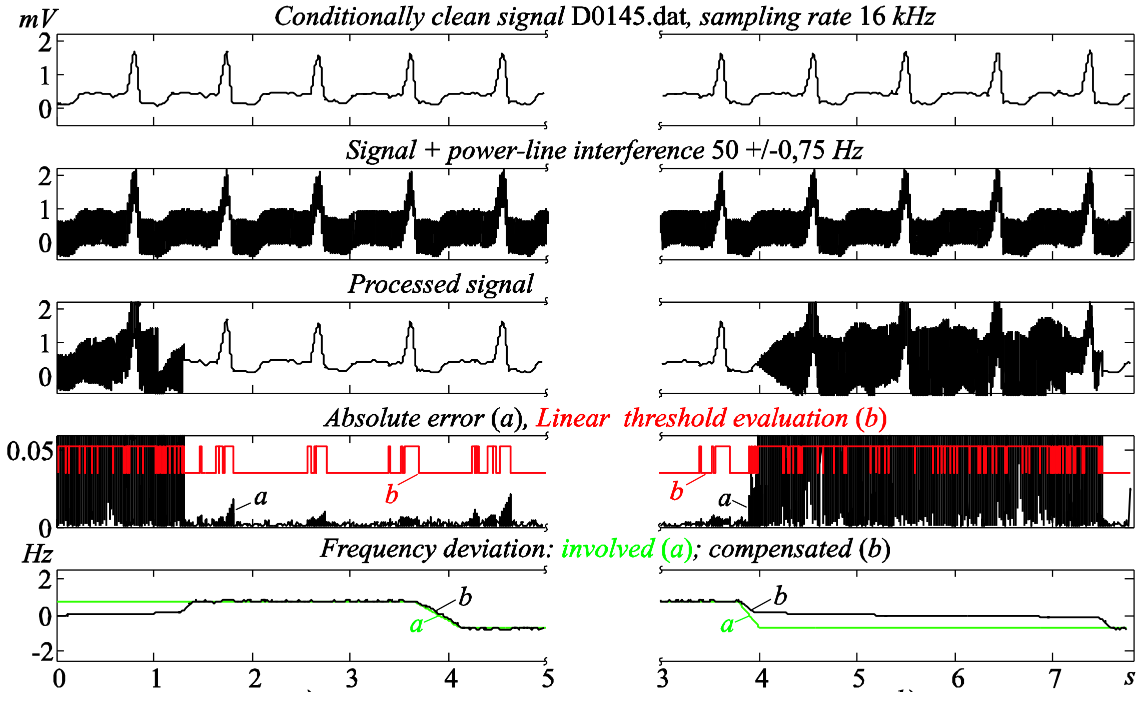
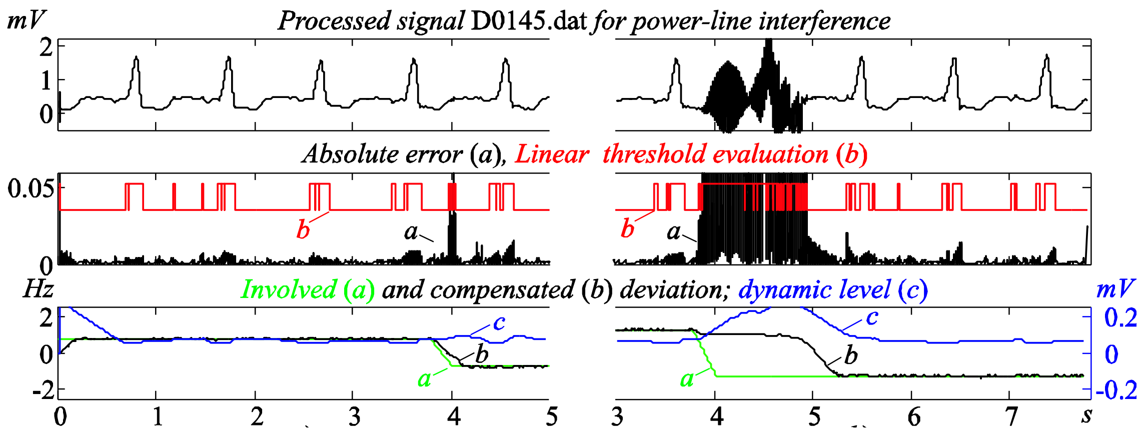


Fig. 4 Experiments in case of Q = 16 kHz and F = 50 ± 0.75 Hz

A modification of the proposed dynamic threshold has been applied in this study. The initial setting is $R_t = 1$. During the frequency compensation, the threshold M_t is currently adjusted multiplying the dynamic R_t by a constant initial threshold value M_{beg}, e.g.

$$M_t = R_t M_{beg}, \quad R_t = \frac{S_{NL}}{S_E}. \tag{14}$$



a) F = 50 ± 0.75 Hz b) F = 50 ± 1.25 Hz

Fig. 5 Experiments with dynamic threshold

The current threshold M_t must be higher or equal to a minimal value $M_t \geq M_{low}$. According to a study in Mihov [7], M_{low} should be no lower than 20 μV. Fig. 5 shows the results of experiments carried out under the same conditions as presented in Fig. 4b except for the dynamic threshold applied (Fig. 4a) and increased mains frequency deviation up to ± 2.5% (Fig. 4b).

The used parameters are $S_E = 0.8$ s, $M_{beg} = 4$ M and $M_{low} = 0.7$ M. The threshold of the sequential linearity criterion is set $M = 70$ μV (corresponding to $M = 100$ μV of the complex linearity criterion). The subtraction procedure starts with threshold of 280 μV, which varies during the experiment between 110 and 50 μV. The advantage of applying the dynamic threshold is obvious especially at abrupt change of the mains frequency. The error in steady state does not exceed 30 μV.

Program realization

The subtraction procedure for PLI removing in case of high sampling rated ECG signals is implemented as function **(PLinterference_removing_Vd2)** in MATLAB environment. The input parameters are:

- **X** – original ECG signal, mV (matrix-row or matrix-column); **Res** – resolution, mV;
- **Q** – sampling rate, Hz; **F** – frequency of the power-line interference, Hz;
- **M** – linearity criterion threshold, mV.

The output parameter is:

- **Y** – filtered (processed) ECG signal, mV (the same size as **X**).

The program code of the function **PLinterference_removing_Vd2** is presented in Fig. 6. The implementation has a built-in compensation of the PLI frequency deviation within ± 2.5%. The initial set of the linearity threshold is reduced down to 70 % since the sequential linearity criterion is applied according to Eq. (2). The complex first **FD**-differences are processed by Eq. (1) with current modification of the coefficient k_d in compliance with Eq. (13). A relative threshold of the *M*-criterion, depending on the ECG signal amplitude and the dynamic threshold level is applied after Eq. (14). If the parameter **M** is set to 0, the relative threshold and the dynamic level are excluded and the procedure is run with an absolute *M*-criterion of 70 μV.

The MATLAB function uses different **K-** and **K**

```
function [Y] = PLinterference_removing_Vd2(X,Res,Q,F,M)

%%% Input parameters:                  %%%       Output parameters:
%%% X - Original signal;               %%%       Y - Processed signal;
%%% Res - Resolution, mV;
%%% Q - Sampling rate, Hz;
%%% F - Interference, Hz;
%%% M - Threshold for D criteria, uV;
                        %%% Initialization %%%
LX = length(X);
dF = F*0.025;
                        %%% Parameters calculating %%%
m=floor(Q/F/2); n=(2*m+1); % Floor 'Multiplicity'
```

```
mmn=floor(Q/(F+dF)/2);
mmx=floor(Q/(F-dF)/2); if mmx==mmn; mmx=mmn+1; end;
r=3; g=floor(Q/F/r); nr=r*g; mr=(r-1)*g/2;
                        %%% Coefficients calculating %%%
KBF0 = sin(nr*pi*F/Q)/sin(pi*g*F/Q)/r; % Initial value of KBF0
KBFmin = sin(nr*pi*(F+dF)/Q)/sin(g*pi*(F+dF)/Q)/r;
KBFmax = sin(nr*pi*(F-dF)/Q)/sin(g*pi*(F-dF)/Q)/r;
KBFspd = (KBFmax-KBFmin)/(4*nr);
KBFnew=KBF0; KBF=KBF0;

K2F0=(sin(n*pi*2*F/Q)/sin(pi*2*F/Q))/n; % Initial value of K2F0
K2Fbeg=(sin(n*pi*2*(F-dF)/Q)/sin(pi*2*(F-dF)/Q))/n;
K2Fend=(sin(n*pi*2*(F+dF)/Q)/sin(pi*2*(F+dF)/Q))/n;
R2k=(K2Fbeg-K2Fend)/(KBFmax-KBFmin);

kd0 = sin(pi*F*2*mmn/Q)/(sin(pi*F*2*mmn/Q)-sin(pi*F*(2*mmx)/Q));
kdbeg=sin(pi*(F-dF)*2*mmn/Q)/(sin(pi*(F-dF)*2*mmn/Q)-sin(pi*(F-dF)*(2*mmx)/Q));
kdend=sin(pi*(F+dF)*2*mmn/Q)/(sin(pi*(F+dF)*2*mmn/Q)-sin(pi*(F+dF)*(2*mmx)/Q));
kd=kd0;
Rdk=(kdbeg-kdend)/(KBFmax-KBFmin);

KF0 = (sin(n*pi*F/Q)/sin(pi*F/Q))/n*cos(0*pi*F/Q);   % Initial value of KF0
KFbeg = (sin(n*pi*(F-dF)/Q)/sin(pi*(F-dF)/Q))/n;
KFend = (sin(n*pi*(F+dF)/Q)/sin(pi*(F+dF)/Q))/n;
KF = KF0;
RKF = (KFbeg-KFend)/(KBFmax-KBFmin);

Y = X;                            % Output buffer definition
A = zeros(1,LX);                  % Amplitude buffer definition
B = zeros(1,LX);                  % Interference buffer definition
B2 = zeros(1,LX);                 % Interference buffer_2 definition

kAmax=0.4/Q*F; kAmin=-kAmax;
M=M*0.7;
Mbeg=4*M; Mlow=0.7*M; Mt=Mbeg; Mnum=0;
Mmin=0; Mmax=1; Mmu=1;
Se=floor(0.8*Q); Sn=ones(1,Se); Snl=Se; l=1;
Bmin=Res; U2=0;

i=1+2*mmx+1;
Yt=0;     % Start of averaging
for j=i-m-1: 1: i+m-1;
    Yt=Yt+X(j)/n;           % Averaging
end                         % End of averaging

                        %%% Algorithm %%%
for i=1+2*mmx+1: 1: LX-2*mmx-1;               % Begin of the Main Loop
    FD1=(X(i+mmx+mmn)-X(i+mmx-mmn))*(1-kd)+(X(i+2*mmx)-X(i))*kd; %FD1 estimation
    FD2=(X(i+mmn)-X(i-mmn))*(1-kd)+(X(i+mmx)-X(i-mmx))*kd;        %FD2 estimation
    Cr = abs(FD1-FD2);    % Linearity estimation

    if M>Bmin;
        if Mmax<X(i); Mmax=X(i); else Mmax=Mmax-(Mmax-X(i))/(2.5*Q); end
        if Mmin>X(i); Mmin=X(i); else Mmin=Mmin+(X(i)-Mmin)/(2.5*Q); end
        Mpp=Mmax-Mmin;
        if Mmu>Mpp; Mmu=Mpp; else Mmu=Mmu+Mpp/(2.5*Q); end;
        Md = Mmu*M; Mbeg=4*Md;
    else Mt=0.07; end

    if Cr<Mt; Mnum=Mnum+1;
        if Mnum<2*mmx+1-mmn; Cr=Mt; else Mnum=Mnum-1; end;
    else Mnum=0; Cr=Mt; end
    Yt=Yt+((X(i+m)-X(i-m-1)))/n; % Averaging
    if Cr < Mt; Dm=0;                    % Linear segment
        Y(i)=X(i)-(X(i)-Yt)/(1-KF);      % Output sample modification
        B(i)=X(i)-Y(i);                  % Interference correction
    else Dm=1;                           % Non-linear segment
        B(i)=B(i-nr)+r*KBF*(B(i-mr)-B(i-mr-g)); % Restoring
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 doi: 10.7546/ijba.2018.22.2.147-158

```
        Y(i)=X(i)-B(i);                   % Output sample estimation
    end
    if abs(B(i-mr)-B(i-mr-g))>Bmin;       % Division zero protection
        KBFnew = (B(i)-B(i-nr))/r/(B(i-mr)-B(i-mr-g));
    end
    if KBFnew-KBF>KBFspd; KBFnew=KBF+KBFspd; end %KF speed protection (rising)
    if KBFnew-KBF<-KBFspd; KBFnew=KBF-KBFspd;end %KF speed protection (falling)
    if KBFnew>KBFmax; KBFnew=KBFmax; end         %KF maximum protection
    if KBFnew<KBFmin; KBFnew=KBFmin; end         %KF minimum protection
    KBF=KBF*0.9+KBFnew*0.1;                      %KF regulation
    KF = KF0+RKF*(KBF-KBF0);                     %KF approximation
    kd = kd0+Rdk*(KBF-KBF0);             % kd approximation
               %%% Begin of Amplitude Deviation Compensation %%%
    B2(i)= B(i)^2;                       % Second Interference buffer
    K2F = K2F0+R2k*(KBF-KBF0);           % K2F approximation
    U2=U2+((B2(i)-B2(i-2*m-1)))/n; % Averaging
    A(i)=1.41*sqrt(abs((U2-B2(i-m)*K2F)/(1-K2F))); % Amplitude estimation
    A(i)=A(i)*1/10+A(i-1)*9/10;          % Amplitude filtration
    if abs(A(i-n))>Bmin;                 % Division zero protection
        kAnew=(A(i)-A(i-n))/A(i-n);      % kA estimation
        if kAnew>kAmax; kAnew=kAmax; end % kA maximum protection
        if kAnew<kAmin; kAnew=kAmin; end % kA minimum protection
    else kA=0; kAnew=0; KBF=KBF0; end;
    kA=kAnew*1/10+kA*9/10;               % kA filtration
    B(i)=B(i)*(1+kA);                    % Amplitude variation compensation
               %%% End of Amplitude Deviation Compensation %%%
    Snl = Snl+Dm-Sn(l); Sn(l)=Dm; l=l+1; if l>Se; l=1; end
    Rt=Snl/Se; Mt=Rt*Mbeg; if Mt<Mlow; Mt=Mlow; end;

end                                      % End of the Function
```

a) Program code

```
...
name='D0145.dat'; Q = 16000; Res = 0.02;
Fp=fopen(name,'r'); X=fread(Fp,'float'); fclose(Fp);
F = 60; M = 0.1;
[Y] = PLinterference_removing_Vd2(X,Res,Q,F,M);
...
```

b) MATLAB fragment with function calling

Fig. 6 The program code of the function **`PLinterference_removing_Vd2`**

The PLI amplitude modulation is compensated according to the published study by Mihov and Badarov [11]. When this amplitude becomes less than 10 μV, the calculation of the mains frequency is stopped. Since the compensation of the PLI amplitude variation takes significant amount of time, it can be removed by deleting or 'commenting' the lines between '**`Begin of Amplitude Deviation Compensation`**' and '**`End of Amplitude Deviation Compensation`**'.

Fig. 7 illustrates the results obtained through an experiment using the 16 kHz sampling rated test signal **`D0145.tst`**, which is additionally mixed by synthesized 50 Hz PLI and sinusoidal amplitude varying from 0 to 1 mV. An abrupt change of the PLI from 50.75 Hz to 40.25 Hz is simulated in the middle of the epoch.

The first subplot shows the test signal, the second one contains the signal processed by the MATLAB function of the subtraction procedure. The third exhibits the extracted power-line interference and the calculated amplitude. The computed mains frequency can be seen in the fourth subplot. The experiment is performed with M = 0.1 mV.

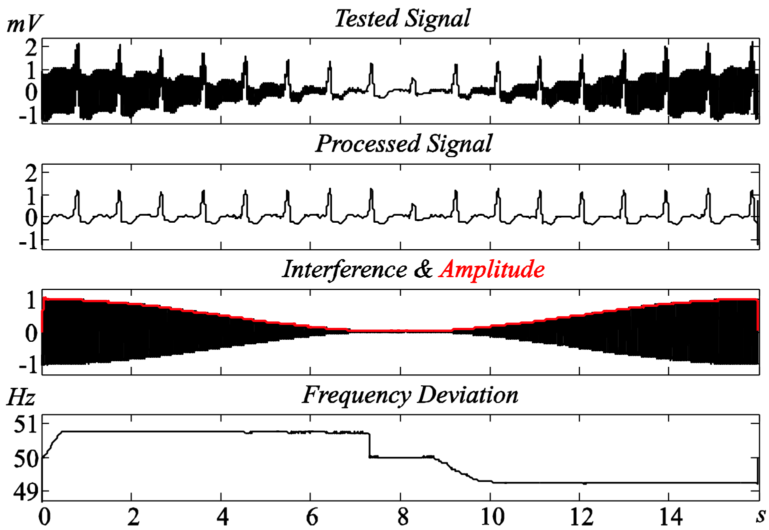

Fig. 7 Experiment with the function **(`PLinterference_removing_Vd2`)**

Conclusion

The basic version of subtraction procedure eliminates successfully the PLI from ECG signals usually sampled with a frequency up to 1 kHz (ratio between sampling rate and mains frequency 20). Recently, a higher over 5 kHz sampling rate is used for specific heart activity analysis (for example to acquire correctly pacemaker's pulses). The sampling rate may reach 128 kHz that corresponds to ratio 2560 between sampling rate and mains frequency.

The carried out study is aimed to developed and introduce appropriate modification of the subtraction procedure intended to overcome the difficulties that involves the high ECG sampling. For this purpose, the following solutions are introduced in the procedure stages:

- Sequential complex linearity criterion is implemented in the Linear segment evaluation;
- Pipe-lined implementation of averaging **K**-filter is applied in the Interference extracting;
- Modified $\mathbf{K_B}$-filter with parameter $r = 3$ is introduced in Interference restoring;
- Effective methodology is proposed to compensate the frequency deviation of the power-line interference. Firstly, the $\mathbf{K_B}$-filter is recalculated, after that the **K**-filter is adequately modified according to established relation;
- Accelerated procedure for the same compensation is adapted to deals with the initial settle and the abrupt change of the mains frequency;
- Program code implementing the subtraction procedure for high sampling rated ECG signals is written in MATLAB.

An analysis of the presented results obtained through experiments show successfully PLI removal from ECG signals with arbitrary ratio between the sampling rate and the mains interference.

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Prof. Georgy Mihov, Eng., Ph.D., D.Sc.
E-mail: gsm@tu-sofia.bg

Georgy Mihov obtained his M.Sc. degree from the Faculty of Radioelectronics, Technical University of Sofia. His Ph.D. thesis (1983) was on the programmable devices for treatment and visualization of ECG. In 2013 he obtained D.Sc. degree on the subtraction procedure for interferences removal from ECG. Since 2007 Georgy Mihov is a Professor with the Technical University of Sofia. His interests are mainly in the field of digital electronics, digital filtration, microprocessor system applications and servicing.