Robust Tracking Control for the Non-isothermal Continuous Stirred Tank Reactor

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Abstract: *A non-isothermal continuous stirred tank reactor (CSTR) is the most important element of chemical industrial equipment which is characterized by a highly nonlinear behavior. It is a multi-input multi-output (MIMO) nonlinear systems exposed to disturbances. The operation of the non-isothermal CSTR can be disturbed by its uncertain parameter such as variation in heat reaction. Therefore, the two difficult problems in CSTR control are tracking trajectory and disturbance attenuation. In this paper, a robust H_∞ fuzzy tracking control via Takagi-Sugeno (T-S) model is designed to robustly stabilize the non-isothermal CSTR system for both concentration and temperature affected by disturbances. T-S fuzzy model approach is proposed to derive the nonlinear model of the CSTR to several local linear models. A parallel distributed compensation control law is used to stabilize the closed-loop system. Linear matrix inequality conditions are derived for analyzing regional robust stability and performance based on Lyapunov function, and an H_∞ criterion is employed to guarantee the attenuation of disturbances. In trajectory tracking framework, an integral action is introduced as a new state variable. Finally, a comparative study between H_∞ controller and linear quadratic regulator (LQR) controller is made. Simulation results show that the proposed H_∞ controller ensures an asymptotic stability and guarantees highly robustness against changes in reaction heat with a good trajectory tracking. The H_∞ controller gives better performance than the classical LQR controller.*

Keywords: *Takagi-Sugeno fuzzy model, Parallel distributed compensation, H-infinity, Linear quadratic regulator, Continuous stirred tank reactor, Bioprocess.*

Introduction

The continuous stirred tank reactor (CSTR) is used in chemical industry for the production of various chemicals and drugs. It exhibits a complicated nonlinear behaviour and uncertain parameters. The non-isothermal CSTR is considered as a one of multi-input multi-output (MIMO) system. Therefore, the problem of controlling the non-isothermal CSTR is an important subject in chemical process and control engineering. Various approaches have been proposed to control the non-isothermal CSTR and to attenuate the influence of its various disturbances [3, 8, 11, 12, 14]. These approaches are directly based on the nonlinear model of the non-isothermal CSTR. Moreover, all this research activities have been carried out for robust controller synthesis of the non-isothermal CSTR using one input to control one output (concentration or temperature).

Several approaches have been developed in order to control a nonlinear system based on the linearity assumption around a given operating point. This method is only locally valid, it limits the domain of application and reduces the performance, due to the nonlinearity of the various components of chemical systems. Recently, in nonlinear control frameworks, Takagi-Sugeno

(T-S) model based fuzzy control approaches are being fast and successfully.

In this work, the Takagi-Sugeno modeling method [10] is used to describe the complex nonlinear model of the non-isothermal CSTR by several simpler sub-models. A robust H_∞ tracking control of nonlinear systems via T-S model is proposed to achieve a good tracking of both desired temperature and concentration with high robustness against changes in reaction heat. A state feedback controller is used for each sub-model, this technique is called parallel distributed compensation (PDC) [13]. The corresponding stability analysis is presented in terms of Lyapunov function. A linear matrix inequality (LMI) conditions are presented to calculate the gain of the controllers. To attenuate the disturbances presented by uncertainty, an H_∞ criterion is considered, and the stability conditions are adapted for perturbed T-S model. An integral control structure is introduced to converge the tracking error to zero in a finite time. Finally, a comparison of the controllers H_∞ and linear quadratic regulator (LQR) is made.

This paper is organized as follows. Section 2 presents the continuous-time T-S fuzzy model for the non-isothermal CSTR. In Section 3, a robust H_∞ tracking control design methodology is proposed. The effectiveness of the proposed method has been verified by simulation results in Section 4. Section 5 presents the design of the LQR controller. Finally, in Section 6, a brief discussion about our results is presented.

T-S fuzzy model of the non-isothermal CSTR system

System description

Let we consider a non-isothermal CSTR with constant volume V, perfect mixing, where a first-order and exothermic reaction $A \rightarrow B$ takes place (Fig. 1).

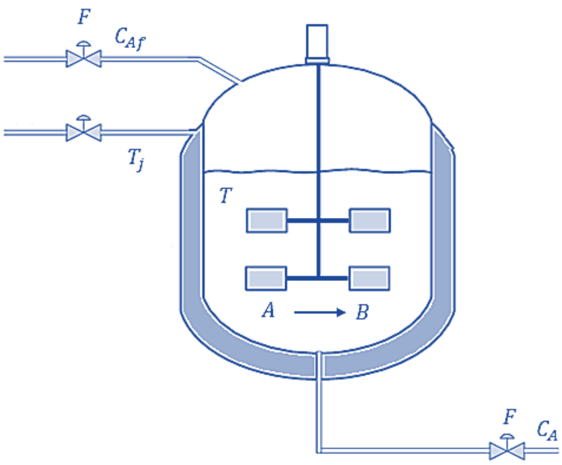

Fig. 1 Non-isothermal continuous stirred-tank reactor

The mathematical model for this process is formulated by carrying out mass and energy balances [6], as follows:

$$\begin{cases} \frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_0 exp\left(\frac{-E}{RT}\right) C_A \\ \frac{dT}{dt} = \frac{F}{V}\left(T_f - T\right) - \frac{\Delta H}{\rho C_p} k_0\, exp\left(\frac{-E}{RT}\right) C_A + \frac{UA}{V\rho C_p}\left(T_j - T\right) \end{cases} . \qquad (1)$$

The parameters values of the CSTR are given in Table 1 [15]. The manipulated inputs of the reactor are the dilution rate F/V and the jacket temperature T_j. The effluent concentration C_A

and the reactor temperature T present the states which will be controlled. ΔH is the reaction heat (also called enthalpy of reaction) presents the change in the enthalpy of a chemical reaction that occurs at a constant pressure.

Table 1. Fixed parameters of the CSTR

Name of the parameter	Symbol and value of the parameter
CSTR volume	$V = 100$ l
Feed concentration	$C_{Af} = 1$ mol·l^{-1}
Feed temperature	$T_f = 350$ K
Activation energy	$E/R = 8750$ K
Rate constant	$k_0 = 7.2 \times 10^{10}$ min^{-1}
Density of reaction	$\rho = 1000$ kg·m^{-3}
Heat capacity	$C_p = 0.239$ J/g·K
Overall heat transfer coefficient	$UA = 5 \times 10^4$ J/min·K

In this work, ΔH is considered as a disturbance because it is an uncertain term and not exactly known. The reaction heat is difficult to measure experimentally. Consequently, a change in ΔH parameter can cause bad stability and poor performance for classical control. We can write Eq. (1) in the state representation form as follows:

$$\begin{bmatrix} \dot{C}_A \\ \dot{T} \end{bmatrix} = \begin{bmatrix} -k_0 exp\left(\frac{-E}{RT}\right) & 0 \\ 0 & -\frac{UA}{V\rho C_p} \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} + \begin{bmatrix} C_{Af} - C_A & 0 \\ T_f - T & \frac{UA}{V\rho C_p} \end{bmatrix} \begin{bmatrix} \frac{F}{V} \\ T_j \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{k_0}{\rho C_p} ex\,p\left(\frac{-E}{RT}\right) C_A \end{bmatrix} \Delta H \quad (2)$$

Takagi-Sugeno model design

Eq. (2) can be described by the following perturbed nonlinear system:

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) + D(x(t))\Phi(\mathrm{t}) \\ y(t) = Cx(t) \end{cases}, \quad (3)$$

where x is the state variable, u is the control input vector, y is the output vector, Φ is the uncertain parameter (disturbance).

Takagi-Sugeno fuzzy model is considered as a multiple model with a common state. The global system behavior is presented as follows [9]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t))[A_i x(t) + B_i u(t) + D_i \Phi(t)] \\ y(t) = \sum_{i=1}^{n} \mu_i(z(t)) C_i \mathrm{X(t)} \end{cases}, \quad (4)$$

where n is the number of local models; A_i, B_i, C_i and D_i are constant matrices that represent a set of linear models; $z(t)$ is the premise variable and μ_i represents the activation degree of a sub-model. These functions have the following properties:

$$\begin{cases} \sum_{i=1}^{n} \mu_i(z(t)) = 1, \forall t \\ 0 \le \mu_i(z(t)) \le 1, \forall\, i = 1, \dots, n \end{cases}. \quad (5)$$

By observing Eq. (2), the fuzzy premise variables are chosen as:

$$\begin{cases} z_1(t) = C_A \\ z_2(t) = T \\ z_3(t) = \exp\left(\frac{-E}{RT}\right) \end{cases}, \tag{6}$$

where $C_A \in [0.7, 1]$ mol·l^{-1} and $T \in [290, 330]$ K.

The maximums and the minimums of the premise variables are noted M_i and m_i respectively, and can be represented by a membership functions F_i and f_i as follows:

$$z_i(t) = F_i(z_i(t))M_i + f_i(z_i(t))m_i, \forall\ i = 1, 2, 3. \tag{7}$$

According to Eq. (5), the membership functions can be obtained by:

$$\begin{cases} F_i(z_i(t)) = \frac{z_i(t) - m_i}{M_i - m_i} \\ f_i(z_i(t)) = \frac{M_i - z_i(t)}{M_i - m_i} \end{cases}, \ \forall\ i = 1, 2, 3. \tag{8}$$

Therefore, we can write the activation functions:

$$\begin{cases} \mu_1(z(t)) = F_1(z_1(t))F_2(z_2(t))F_3(z_3(t)) \\ \mu_2(z(t)) = f_1(z_1(t))F_2(z_2(t))F_3(z_3(t)) \\ \mu_3(z(t)) = F_1(z_1(t))f_2(z_2(t))F_3(z_3(t)) \\ \mu_4(z(t)) = f_1(z_1(t))f_2(z_2(t))F_3(z_3(t)) \\ \mu_5(z(t)) = F_1(z_1(t))F_2(z_2(t))f_3(z_3(t)) \\ \mu_6(z(t)) = f_1(z_1(t))F_2(z_2(t))f_3(z_3(t)) \\ \mu_7(z(t)) = F_1(z_1(t))f_2(z_2(t))f_3(z_3(t)) \\ \mu_8(z(t)) = f_1(z_1(t))f_2(z_2(t))f_3(z_3(t)). \end{cases} \tag{9}$$

The global T-S model is then inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i(z(t))[A_i x(t) + B_i u(t) + D_i \Phi(t)] \\ y(t) = Cx(t) \end{cases}, \tag{10}$$

where

$$A_i = \begin{bmatrix} -k_0 M_3 & 0 \\ 0 & -\frac{UA}{V\rho C_p} \end{bmatrix}, \forall\ i = 1, \ldots, 4;\ A_i = \begin{bmatrix} -k_0 m_3 & 0 \\ 0 & -\frac{UA}{V\rho C_p} \end{bmatrix}, \forall\ i = 5, \ldots, 8;$$

$$B_1 = B_5 = \begin{bmatrix} C_{Af} - M_1 & 0 \\ T_f - M_2 & \frac{UA}{V\rho C_p} \end{bmatrix};\ B_2 = B_6 = \begin{bmatrix} C_{Af} - m_1 & 0 \\ T_f - M_2 & \frac{UA}{V\rho C_p} \end{bmatrix};\ B_3 = B_7 = \begin{bmatrix} C_{Af} - M_1 & 0 \\ T_f - m_2 & \frac{UA}{V\rho C_p} \end{bmatrix};$$

$$B_4 = B_8 = \begin{bmatrix} C_{Af} - m_1 & 0 \\ T_f - m_2 & \frac{UA}{V\rho C_p} \end{bmatrix};\ D_1 = D_3 = \begin{bmatrix} 0 \\ -\frac{k_0}{\rho C_p} M_1 M_3 \end{bmatrix};\ D_2 = D_4 = \begin{bmatrix} 0 \\ -\frac{k_0}{\rho C_p} m_1 M_3 \end{bmatrix};$$

$$D_5 = D_7 = \begin{bmatrix} 0 \\ -\frac{k_0}{\rho C_p} M_1 m_3 \end{bmatrix};\ D_6 = D_8 = \begin{bmatrix} 0 \\ -\frac{k_0}{\rho C_p} m_1 m_3 \end{bmatrix};\ \text{and } C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Non-isothermal CSTR T-S model validation

To validate the synthesized Takagi-Sugeno model, we simulate the T-S multi-model and the nonlinear model of the non-isothermal CSTR in parallel and we compare the resulting curves. We maintain a constant value of ΔH = -5.10^4 J/min·K.

Figs. 2 and 3 present the simulation results.

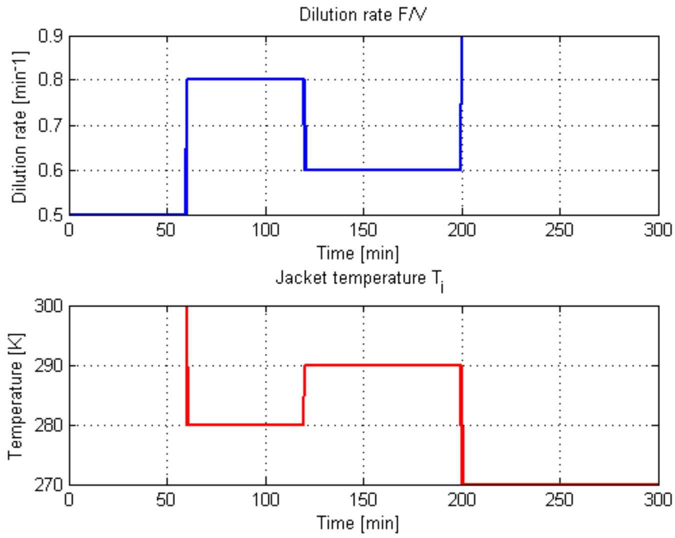

Fig. 2 Inputs signal

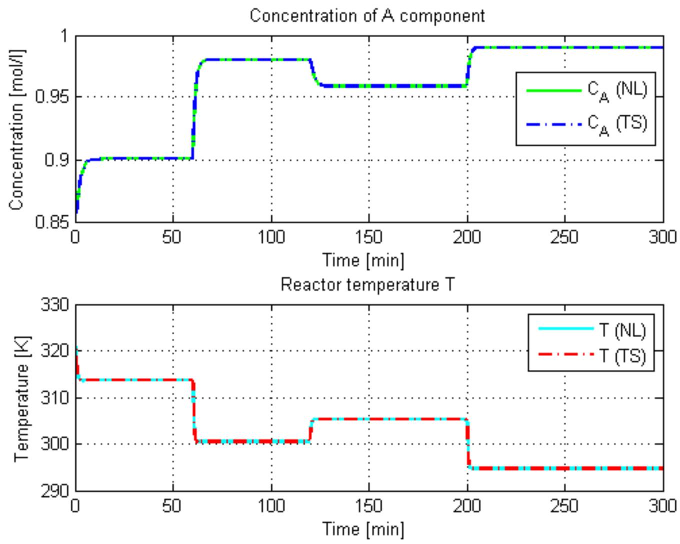

Fig. 3 Comparison of the NonLinear system (NL) and Takagi-Sugeno model (TS)

We show the resemblance between the outputs of the T-S model and the nonlinear model of the non-isothermal CSTR. These results prove the quality of the approximation of a nonlinear system by a T-S multi-model.

Robust H_∞ tracking control synthesis

Trajectory tracking

The objective is to track asymptotically a reference trajectory by the system output. Let $y_r(t)$ denote the desired signal for the output $y(t)$ to track asymptotically. To solve tracking problems, the integrator of the tracking error e_{Ti} is introduced as a new system state defined as:

$$e_{Ti} = \int e_T(t)\,dt = \int (y_r(t) - y(t))\,dt. \quad (12)$$

Then, the external perturbed T-S model which augmented by tracking error can be written as:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{n} \mu_i(z(t))[\bar{A}_i X(t) + \bar{B}_i u(t) + \bar{D}_\iota \bar{\Phi}] \\ Y(t) = \bar{C} X(t) \end{cases}, \quad (13)$$

where

$$X(t) = \begin{bmatrix} x(t) \\ e_{Ti}(t) \end{bmatrix};\ \bar{A}_i = \begin{bmatrix} A_i & 0 \\ -C & 0 \end{bmatrix};\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix};\ \bar{D}_\iota = \begin{bmatrix} D_i & 0 \\ 0 & I \end{bmatrix};\ \bar{\Phi} = \begin{bmatrix} \Phi \\ y_r \end{bmatrix};\ \bar{C} = [C \quad 0].$$

H_∞ tracking controller design

H_∞ control is the most significant scheme to efficiently eliminate the effect of disturbances on the perturbed systems.

A robust H_∞ tracking control based on T-S multi-model approach is carried out via the so-called PDC controller with H_∞ criteria optimization to ensure high performance requirements and high robustness.

The fuzzy PDC controller uses the same fuzzy sets of T-S model, and it can be expressed as [5]:

$$u(t) = -\sum_{i=1}^{n} \mu_i(z(t)) K_i X(t). \quad (14)$$

where K_i are the gains of feedback controllers, n is the number of local models.

By combining the control law (14) and the system (13), the closed-loop system becomes:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t))[G_{ij} X(t) + \bar{D}_\iota \bar{\Phi}] \\ Y(t) = \bar{C} X(t) \end{cases}, \quad (15)$$

with $G_{ij} = \bar{A}_\mathrm{i} - \bar{B}_\mathrm{i} K_j$.

Now, the goal is to find the gains K_i that guarantee the stability of the system (15) and ensures the robustness of the fuzzy model system against disturbances Φ.

Let us consider the following H_∞ control performance [4]:

$$\int_0^\infty x^\mathrm{T}(t) Q x(t) dt < \rho^2 \int_0^\infty \bar{\Phi}^\mathrm{T}(t) \bar{\Phi}(t) dt \quad (16)$$

with Q is a positive definite matrix and ρ is a scalar performance level to be minimized to ensure the best dynamics of the closed-loop system. In general, ρ is chosen as a positive small value less than 1.

Theorem 1. The closed-loop perturbed system (15) is quadratically stable if there exists a matrix $P = P^\mathrm{T} > 0$, a positive constant η and feedback gains K_i that satisfy the following conditions:

$$Q > 0; \quad (17.1)$$

$$\Psi_{ii} + \eta^{-1} P \bar{D}_i \bar{D}_i^{\mathrm{T}} P + Q < 0, i = 1, 2, \ldots, n; \tag{17.2}$$

$$\Psi_{ij} + \Psi_{ji} + \eta^{-1} P (\bar{D}_i \bar{D}_j^{\mathrm{T}} + \bar{D}_j \bar{D}_i^{\mathrm{T}}) P + 2Q < 0, \ \ i < j; \tag{17.3}$$

with $\Psi_{ij} = (\bar{A}_{\mathrm{i}} - \bar{B}_{\mathrm{i}} K_j)^{\mathrm{T}} P + P(\bar{A}_{\mathrm{i}} - \bar{B}_{\mathrm{i}} K_j)$.

Proof: We choose a Lyapunov function for the system (15) as:

$$V(x) = X^{\mathrm{T}} P X \text{ and } \dot{V} = \dot{X}^{\mathrm{T}} P X + X^{\mathrm{T}} P \dot{X}, \text{ with } P = P^{\mathrm{T}} > 0. \tag{18}$$

If the system (15) can satisfy the Lyapunov condition, the stability of the closed-loop system can be guaranteed. By substituting Eq. (15) to Eq. (18), we get:

$$\dot{V} = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t)) X^{\mathrm{T}} P(\bar{A}_{\mathrm{i}} - \bar{B}_{\mathrm{i}} K_j) + (\bar{A}_{\mathrm{i}} - \bar{B}_{\mathrm{i}} K_j)^{\mathrm{T}} P X + \\ + X^{\mathrm{T}} P \bar{D}_i \bar{\Phi} + \bar{\Phi}^{\mathrm{T}} \bar{D}_i^{\mathrm{T}} P X < 0. \tag{19}$$

Lemma 1. For real matrices R and S with the appropriate dimensions and a positive constant η, the following inequalities hold:

$$R^{\mathrm{T}} S + R^{\mathrm{T}} S \leq \eta R^{\mathrm{T}} R + \eta^{-1} S^{\mathrm{T}} S. \tag{20}$$

By applying the *Lemma 1* in Eq. (19), we get:

$$\dot{V} \leq \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t)) X^{\mathrm{T}} [\Psi_{ij} + \eta^{-1} P \bar{D}_i \bar{D}_i^{\mathrm{T}} P] X + \eta \bar{\Phi}^{\mathrm{T}} \bar{\Phi} < 0. \tag{21}$$

By using the H_∞ criterion Eq. (16) in Eq. (21), the stability conditions are verified if:

$$\dot{V} \leq \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t)) X^{\mathrm{T}} [\Psi_{ij} + \eta^{-1} P \bar{D}_i \bar{D}_i^{\mathrm{T}} P + Q] X < 0 \tag{22}$$

with $\eta = \rho^2$

Inequality Eq. (22) can be rewritten as:

$$\dot{V} \leq \sum_{i=1}^{n} \mu_i^2(z(t)) [\Psi_{ii} + \eta^{-1} P \bar{D}_i \bar{D}_i^{\mathrm{T}} P + Q] + \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(z(t)) \mu_j(z(t)) [\tfrac{\Psi_{ij} + \Psi_{ji}}{2} + \\ + \frac{\eta^{-1} P(\bar{D}_i \bar{D}_j^{\mathrm{T}} + \bar{D}_j \bar{D}_i^{\mathrm{T}}) P}{2} + Q] < 0. \tag{23}$$

Hence, the stability conditions becomes:

$$\Psi_{ii} + \eta^{-1} P \bar{D}_i \bar{D}_i^{\mathrm{T}} P + Q < 0, \ \ i = 1, 2, \ldots, n;$$

$$\Psi_{ij} + \Psi_{ji} + \eta^{-1} P (\bar{D}_i \bar{D}_j^{\mathrm{T}} + \bar{D}_j \bar{D}_i^{\mathrm{T}}) P + 2Q < 0, \ \ i < j.$$

The inequalities of this theorem are not LMI. Consequently, it is complex to obtain the solutions with classical convex optimization algorithms. So, we consider a new variable $N = P^{-1}$ and the convenient bijective change of variable $M_i = K_i N$. Then, the stable H_∞ tracking controller design problem is given in the following inequalities:

$$N\bar{A}_i^T + \bar{A}_i N - M_i^T \bar{B}_i^T - \bar{B}_i M_i + \eta^{-1} \bar{D}_i \bar{D}_i^T + NQN < 0,\ i = 1, \dots, n \tag{24.1}$$

$$N\bar{A}_i^T + \bar{A}_i N + N\bar{A}_j^T + \bar{A}_j N - M_j^T \bar{B}_i^T - \bar{B}_i M_j - M_i^T \bar{B}_j^T - \bar{B}_j M_i + \eta^{-1}\left(\bar{D}_i \bar{D}_j^T + \bar{D}_j \bar{D}_i^T\right) + 2NQN \le 0,\ i < j \le n. \tag{24.2}$$

For achieving the control input constraint, the following Theorem 2 provides the LMI conditions such that:

Theorem 2. We consider that the initial condition $x(0)$ is known. The input constraint $\|u(k)\|_2 \le \mu$ is enforced at all time $k \ge 0$ if the following LMI conditions are satisfied [2].

$$\begin{bmatrix} 1 & x(0)^T \\ x(0) & N \end{bmatrix} \ge 0, \tag{25.1}$$

$$\begin{bmatrix} N & M_i^T \\ M_i & \mu^2 I \end{bmatrix} \ge 0, \tag{25.2}$$

where $N = P^{-1}$ and $M_i = K_i N$.

Linear quadratic regulator controller design

The optimal linear quadratic regulator (LQR) control is presented in [1, 7]. It consist of reducing the performance index to a minimize value in order to achieving acceptable performance of the system. The performance index J is given by:

$$J = \int_0^\infty (X(t)^T Q X(t) + u(t)^T R u(t)) dt, \tag{26}$$

where $Q \ge 0$ is a symmetric state weighting matrix, and $R > 0$ is symmetric control weighting matrix. The LQR control law is defined as:

$$u = -KX, \tag{27}$$

where K is the optimal feedback gain matrix.

Now, the goal is to find the gain matrices K_i that minimizes the performance index under the constraints of Q and R matrices. This can be achieved using the following equation:

$$\bar{A}_i^T P_i + P_i \bar{A}_i - P_i \bar{B}_i R^{-1} \bar{B}_i^T P_i + Q = 0 \tag{28}$$

then

$$K_i = R^{-1} \bar{B}_i^T P_i. \tag{29}$$

Simulations results

The design of the proposed H_∞ controller requires the calculation of different feedback gains K_i of each local model, by satisfying the LMI conditions developed previously.

The constraints for control signal: jacket temperature $\in$ [280, 320] K, setting the initial condition as $x(0) = [0.6\ 335.6]^T$, dilution rate $\in$ [0, 0.5] min^{-1}.

The following feedback gains of the H_∞ controller are obtained:

$$K_1 = \begin{bmatrix} 42.9608 & 0.2571 & -7.4432 & -0.1231 \\ -821.4350 & -5.2213 & 142.3175 & 1.5438 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 42.9447 & 0.2571 & -7.4404 & -0.1231 \\ -821.6038 & -5.2213 & 142.3418 & 1.5439 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 42.9608 & 0.2632 & -7.4432 & -0.1260 \\ -1232.1402 & -7.8545 & 213,4735 & 2.8051 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 42.9447 & 0.2632 & -7.4404 & -0.1261 \\ -123.2261 & -7.8544 & 213.4898 & 2.8051 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 42.9617 & 0.2571 & -7.4432 & -0.1231 \\ -821.4549 & -5.2213 & 142.3186 & 1.5438 \end{bmatrix},$$

$$K_6 = \begin{bmatrix} 42.9455 & 0.2571 & -7.4405 & -0.1231 \\ -821.6270 & -5.2213 & 142.3432 & 1.5439 \end{bmatrix},$$

$$K_7 = \begin{bmatrix} 42.9617 & 0.2632 & -7.4432 & -0.1261 \\ -1232.1616 & -7.8545 & 213.4747 & 2.8051 \end{bmatrix},$$

$$K_8 = \begin{bmatrix} 42.9455 & 0.2632 & -7.4405 & -0.1261 \\ -1232.2797 & -7.8545 & 213.4904 & 2.8051 \end{bmatrix}.$$

The simulation results are depicted in the Figs. 4-8.

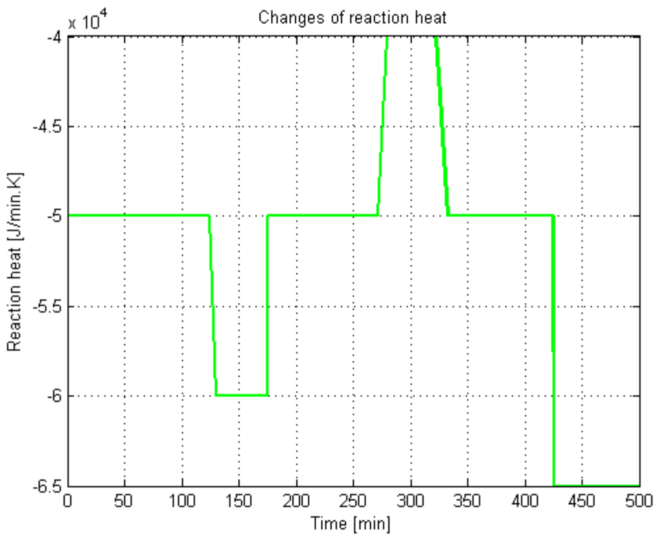

Fig. 4 Changes of reaction heat

The simulation results show that the input constraints controlled by controllers H_∞ and LQR are both achieved. It can be seen from Figs. 7 and 8 that the proposed H_∞ controller has the satisfactory concentration and temperature tracking performance better than that of LQR controller. Therefore, the simulation results demonstrate that the proposed H_∞ tracking control could guarantee the robust asymptotic stability of the non-isothermal CSTR in the event of changes in heat reaction.

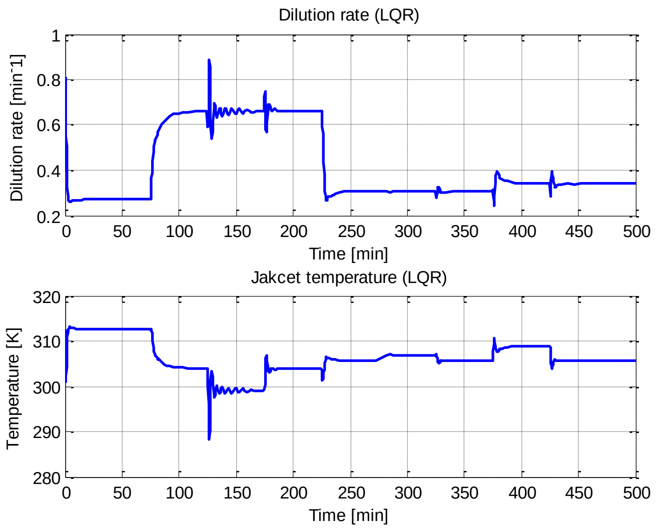

Fig. 5 Control inputs of the LQR controller

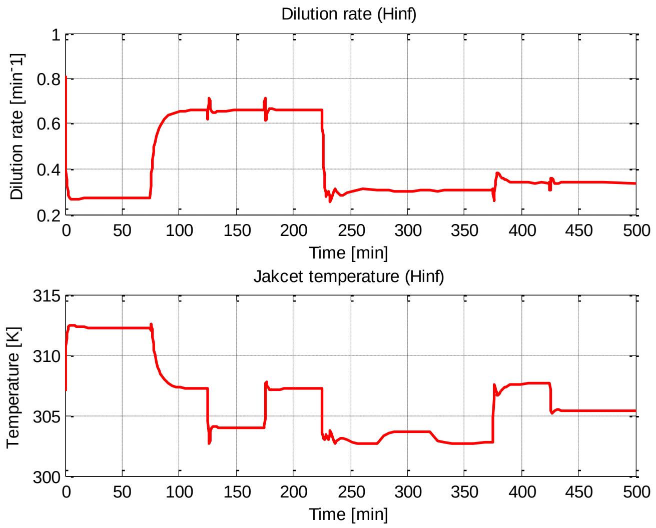

Fig. 6 Control inputs of the H_∞ controller

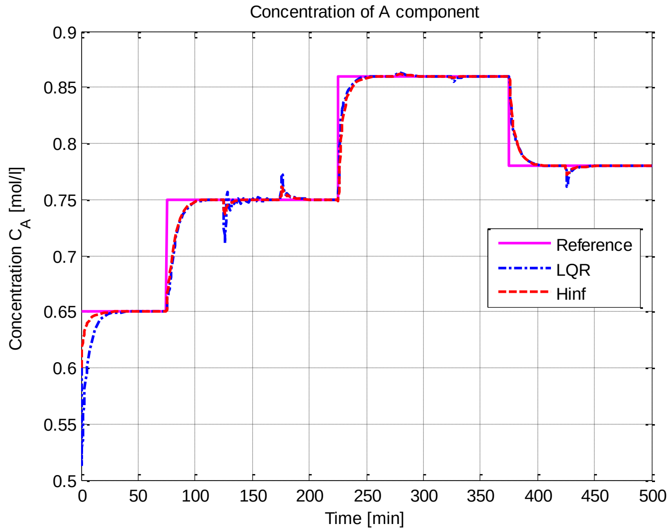

Fig. 7 Tracking response for concentration

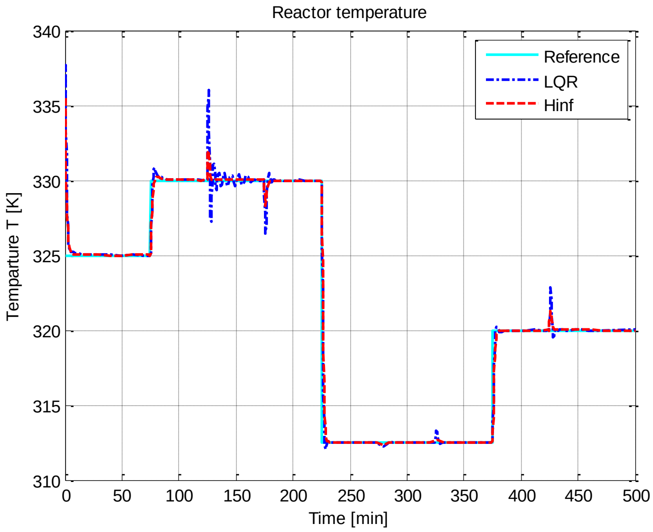

Fig. 8 Tracking response for temperature

Conclusion

This paper concerns the design of a robust H_∞ tracking controller via T-S multi-model for a nonlinear non-isothermal CSTR witch exposed to disturbances. Firstly, the CSTR is modelled in a continuous form of Takagi-Sugeno system. Then the H_∞ tracking controller is synthesized based on a PDC technique. To track a desired temperature and concentration of the non-isothermal reactor, an integral action is introduced for the tracking error and an augmented system is obtained. To attenuate the effects of heat reaction changes, an H_∞ criterion is considered, and the design conditions for quadratic stability of the T-S model are formulated as an LMI. Finally, a comparative study showed that the H_∞ controller provides a better performance of the reference tracking and disturbance attenuation than the LQR controller.

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